Using local observations of the geomagnetic field to improve crustal field estimates from global models

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The Earth’s magnetic field

• Most of the field is from the **Earth’s core**
  − varies slowly with time (*months to years*)
• Local fields from magnetized rocks in **Earth’s crust**
  − relatively **stable** with time
• Fields due to currents in the **ionosphere and magnetosphere**
  − variations from **seconds to years**
Reconstructing the magnetic field vector at the drill site

\[ B = B_{\text{main}}(r, t) + B_{\text{external}}(r, t) + B_{\text{crust}}(r) \]
Sources and errors

Reference field vector for drilling = $B + \varepsilon$

1. Ideally, account for all sources

$B_1 = B_{\text{main}} + B_{\text{crust}} + B_{\text{external}}$

$\varepsilon_1 = \varepsilon_{\text{main}} + \varepsilon_{\text{crust}} + \varepsilon_{\text{external}}$

2. If external fields are ignored

$B_2 = B_{\text{main}} + B_{\text{crust}} + 0$

$\varepsilon_2 = \varepsilon_{\text{main}} + \varepsilon_{\text{crust}} + B_{\text{external}}$

3. If crustal and external fields are ignored

$B_3 = B_{\text{main}} + 0 + 0$

$\varepsilon_3 = \varepsilon_{\text{main}} + B_{\text{crust}} + B_{\text{external}}$
All global main field models capture some of the crustal field...

- Novel weighting methods applied to satellite data
- Vector data at all latitudes
- Low-noise lithospheric field model
- Piecewise linear SV
- External dipole magnetic field with VMD index rapid time-dependence

…but local observations in vicinity of drilling site complete the picture

- Direct measurements of the vector field
  - on land
  - at sea

- Direct measurements of the scalar field
  - inversions for source properties followed by forward modelling
  - transformations
Direct measurements of the vector field on land
Direct measurements of the vector field at sea

**Platform:** The Adventurer - holder of the record for the fastest circumnavigation of the globe – reasonably non-magnetic

**Instruments:** Vector and scalar magnetometers, ring-laser gyro and GPS

A collaborative project between Tech21 and BGS
Typical marine vector survey

10 km by 10 km
Direct measurements of the scalar field

image courtesy of Sander Geophysics

image courtesy of Fugro

image courtesy of PGS
Typical aeromagnetic survey

100 km by 100 km
Aeromagnetic data processing

- raw data
  - compensate for magnetic effect of aircraft
    - compensated data
  - Base station data
    - remove time-varying external field
      - diurnally corrected data
  - IGRF values
    - remove reference field
      - diurnal and IGRF corrected data
  - remove cultural noise
    - culturally corrected data
  - network level
    - levelled data
  - micro-level
    - micro-levelled data
  - tie with neighbouring surveys
    - final data

- data channel
  - processing step
  - Model

sometimes some channels are missing
model not always specified
Validation of scalar data for gross errors, noise content and absolute level

- check coordinates
- check base station data
- check model
- check processing e.g. compare data channels
- compare with independent data
- downward and upward continuation
Assumptions with scalar data

Scalar magnetometer measures $|B_o|$
Total intensity anomaly **defined** as $\Delta F = |B_o| - |B_m|$
This is **not** the same as $|B_c|$

$B_m = (X_m, Y_m, Z_m)$ estimated from a global model

If crustal field is small compared to main field (200 nT cf 50000 nT), $\Delta F$ is well approximated by the projection of crustal field vector onto the main field vector

$$\Delta F \approx \frac{(X_c X_m + Y_c Y_m + Z_c Z_m)}{F_m}$$

**equation (1)**
Inversions of scalar data

\[ V(P) = \int_R M(Q) \psi(P, Q) \, dv \]

Magnetic rock (region R)

Magnetisation at point Q

Geometrical function relating points P and Q

Magnetic potential at point P (outside R)

region containing magnetic sources

Inverse problem

Forward problem
Inversions of scalar data

- Assume magnetisation induced by main field
- Assume magnetisation does not vary with depth
- Determine top surface of R from seismic data

Observed anomaly, $\Delta F$

Seismically-determined depth to magnetic basement
Transformations of scalar data
Applications of Fourier transformation techniques

**IN:** F anomalies at surface
**OUT:** D and I anomalies at surface
D, I and F anomalies at depth
Transformations of scalar data
Applications of Fourier transformation techniques

F anomalies at surface

D anomalies at surface and at depth 4 km
Scalar to vector transformations

$\mathbf{B}_c = (X_c, Y_c, Z_c)$ is the gradient of a scalar potential $V_c$ which satisfies Laplace’s equation $\nabla^2 V_c = 0$

A solution to Laplace’s equation is:

$$V_c(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{2\pi i (ux + vy) + z\sqrt{u^2 + v^2}} A(u, v) dudv$$

$\Delta F$ also satisfies Laplace’s equation and can be written as (assuming data collected at constant altitude):

$$\Delta F(x, y, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{2\pi i (ux + vy)} C(u, v) dudv$$

Use equation (1) linking $\Delta F$ and $\mathbf{B}_c$ to get an expression for $A(u, v)$ in terms of $C(u, v)$

Fewer assumptions about the geometrical or magnetic properties of the sources than with inversions
Downward continuation

\[ \tilde{\Phi}(u, v, z) = \Lambda_{uv} \tilde{\Phi}(u, v, z_0) \]

\[ \Lambda_{uv} = \exp(2\pi \sqrt{u^2 + v^2} \Delta z) \]

Small error in \( \tilde{\Phi}(u, v, z_0) \) with large \( u, v \) (short wavelengths) results in large errors in \( \tilde{\Phi}(u, v, z) \). Consequence is high amplitude and short wavelength noise in resulting anomalies.

Equivalent filter operator but with damping (parameter \( \lambda \)):

\[ \Lambda_{uv} = \frac{\exp(-2\pi \sqrt{u^2 + v^2} \Delta z)}{\exp(-4\pi \sqrt{u^2 + v^2} \Delta z) + \lambda \left(2\pi \sqrt{u^2 + v^2}\right)^4} \]
Validation of downward continuation

Compare damped downward-continued anomalies which are then upward-continued, with input data.

- survey boundary
- differences in drilling area small
- sampling noise
- depth 4 km
- differences in nT
- high gradients
BP Miller field - small F anomaly does not mean small D anomaly

F anomalies at surface

D anomalies at surface and at depth 4 km
BP Miller field – D anomalies from marine vector survey agree
### Downward continuation

BP Miller – effect of downward continuation

4 km ~ max drilling depth

<table>
<thead>
<tr>
<th></th>
<th>declination</th>
<th>inclination</th>
<th>total intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>surface</td>
<td>-0.497</td>
<td>-0.035</td>
<td>-56.2</td>
</tr>
<tr>
<td>depth 4 km</td>
<td>-0.751</td>
<td>-0.026</td>
<td>-68.6</td>
</tr>
<tr>
<td>difference</td>
<td><strong>0.254</strong></td>
<td><strong>-0.009</strong></td>
<td><strong>12.4</strong></td>
</tr>
</tbody>
</table>

(declination and inclination in degrees, total intensity in nT)
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Estimating $\varepsilon_{\text{main}} + \varepsilon_{\text{crust}}$
Confidence levels

• Error distributions are not usually normal

• Should not use multiples of $\sigma$ and assume same confidence as with a normal distribution

• Confidence levels relevant for any error distribution

• Uncertainties presented as limits for confidence levels…
  - 68.3% (equivalent to $1\sigma$ if normal)
  - 95.4% (equivalent to $2\sigma$ if normal)
  - 99.7% (equivalent to $3\sigma$ if normal)
\[ B_2 = B_{\text{main}} + B_{\text{crust}} + 0 \]

\[ \varepsilon_2 = \varepsilon_{\text{main}} + \varepsilon_{\text{crust}} + B_{\text{external}} \]

95.4% confidence limit

\begin{tabular}{|c|c|c|}
\hline
D & I & F \\
\hline
0.26° & 0.12° & 73 nT \\
\hline
\end{tabular}
Conclusions

• The crustal field $B_{\text{crust}}$ represents an offset error to the geomagnetic field vector from a global model

• Local magnetic observations are necessary to determine $B_{\text{crust}}$ and reduce errors

• Further improvement in estimates of $B$ are possible with use of real-time magnetic data for external field $B_{\text{external}}$
Acknowledgements

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