Definition of the ISCWSA Error Model

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</tbody>
</table>
1 Scope

This document details the mathematical framework underpinning the ISCWSA error model for wellbore positioning. The aim is to define the current version of the error model mathematics in one concise document and as such, it brings together material that was previously available in a number of SPE papers and ISCWSA documents. This document is intended for implementers and those who wish to understand the details of the model rather than for users of the model’s results. A familiarity with the basic concepts of borehole surveying is assumed.

The document is broken down into twelve sections.

Firstly there is an introduction and overview of the constituent elements of the ISCWSA error model and some comments on what the model does and does not include. Secondly the derivation of the error model mathematics is described. There then follows some guidance for implementers which summarises the core model section. Then particular details of the MWD and gyro models are discussed. Finally, the ISCWSA test wells are specified.
2 Table of Contents

1 Scope .............................................................................................................................................. 4
2 Table of Contents ................................................................................................................................. 5
3 Background ......................................................................................................................................... 7

3.1 Overview of the Error Model ............................................................................................................ 9
3.2 Assumptions and Limitations of the Model ...................................................................................... 13
4 Details of the Mathematical Framework .............................................................................................15

4.1 Definition of Axes ............................................................................................................................ 15
4.2 Notation Used in the Mathematical Framework .............................................................................. 16
4.3 Notation Used in the Weighting Functions .................................................................................... 17

4.3.1 Note on the use of Azimuth ........................................................................................................ 17
4.4 Evaluation of Position Uncertainty .................................................................................................. 19
4.5 Derivation of Weighting Functions .................................................................................................. 23
4.6 Singular Weighting Functions .......................................................................................................... 24
4.7 Summation of Uncertainty Terms and Propagation Modes ............................................................ 25
4.8 Transformation to Borehole Axes ................................................................................................... 28
4.9 Position Uncertainty Model for a Specific Tool ............................................................................. 29

5 Error Sources and Weighting Functions .............................................................................................30

5.1 Common Elements of Modelling .....................................................................................................30

5.1.1 Depth Terms ............................................................................................................................... 30
5.1.2 Borehole Misalignments ............................................................................................................ 30

6 MWD Modelling ..................................................................................................................................32

6.1 MWD Revision 4 Position Uncertainty Models ..............................................................................32

6.2 Weighting Functions ........................................................................................................................ 33

6.2.1 Sensor Terms ............................................................................................................................. 33
6.2.2 Drillstring Interference .............................................................................................................. 33
6.2.3 Geo-magnetic Reference ........................................................................................................... 34

6.3 History of the MWD Error Model ................................................................................................... 35

6.3.1 Revision 0 ................................................................................................................................... 35
6.3.2 Revision 1 ................................................................................................................................... 35
6.3.3 Revision 2 ................................................................................................................................... 36
6.3.4 Revision 3 ................................................................................................................................... 36
6.3.5 Revision 4 ................................................................................................................................... 36
### 6.3.6 Bias Models

#### 7 Gyro Models

- 7.1 Sensor Configuration
- 7.2 Operating Modes

#### 8 Utility Models

- 8.1 Inclination Only Surveys
- 8.2 CNI and CNA
- 8.3 Testing and Validation

#### 9 Implementation

- 9.1 Inputs
- 9.2 Output
- 9.3 Software Flow
- 9.4 Confidence Level
- 9.5 Example Implementation

#### 10 References

#### 11 List of Error Sources and Weighting Functions

- 11.1 Error Sources Common to Both Gyro and MWD Models
- 11.2 MWD Error Sources
- 11.3 Gyro Error Sources
- 11.4 Utility Sources
- 11.5 Vertical Singularities
- 11.6 Historic Terms: No Longer Used in the MWD Model After Revisions 3
3 Background

Like all measurements, borehole surveys are subject to errors and uncertainties which mean that a downhole survey result is not 100% accurate. For many applications, such as anti-collision and target sizing, it is very important to be able to quantify the uncertainty in position along a wellbore. However, since many different factors contribute to the final position uncertainty, determining these bounds is not a trivial matter.

The Industry Steering Committee for Wellbore Survey Accuracy (ISCWSA) (also known as the SPE Wellbore Positioning Technical Section) has developed an error model in an attempt to quantify the accuracy or uncertainty of downhole surveys. This error model consists of a body of mathematics for evaluating the uncertainty envelope around the survey. The aim is to provide a method of evaluating well bore position uncertainties based on a standardised and generalised set of equations, which will cover most scenarios and which can be implemented in a consistent manner in well planning and directional software.

The model starts from identified physical phenomena which contribute to survey errors, and then evaluates how these phenomena effect the survey measurements at each station and how these errors then build up along a survey leg and ultimately along the entire wellbore. Typically the mathematics are implemented in directional drilling software in which the user selects the appropriate tool model for use, along with the wellbore surveys or plan in order to obtain an uncertainty or anti-collision report.

The initial version of the model covered MWD surveys and was described in detail in a SPE paper [1]. This work was later extended with the publication of a gyro model [2] and a depth error paper [3]. There have also been subsequent revisions and corrections of the error models (see section 6.2). This document sets out to define the current version of the error model. The reader looking for further details should consult the original papers. Those seeking a more general introduction to the principles and practises of borehole surveying are referred to the online e-book [9].

Changes to the error model are discussed and agreed via the ISCWSA Error Model Maintenance Committee. This is an industry wide workgroup and, by prior agreement with the chairman, attendance is open to anyone who wishes to contribute to the development of the model. See [http://www.iscwsa.net/index.php/workgroups/model-management/](http://www.iscwsa.net/index.php/workgroups/model-management/) for more details, including minutes of the latest meetings.

The model may be considered to comprise of two parts; firstly the underlying algorithmic framework which provides all the mathematical building blocks needed to evaluate and accumulate uncertainties for any possible tool, and secondly the details required to model a specific tool. These details are normally defined in what are variously called an Instrument Performance Model, Position Uncertainty Model, IPM file, tool code or error model. In this document we will use the term Position Uncertainty Model abbreviated to PUM.
The Error Model Maintenance committee is mainly concerned with the algorithms. ISCWSA’s position is that tool providers are best placed to make use of this framework to define the PUM to model a specific tool. There are two exceptions to this rule –

i) since many MWD tools are similar in performance and limited more by environmental considerations, the Error Model Maintenance committee defines the PUMs for a generic MWD tool model which comes in eight variants (standard MWD/axial correction, fixed/floating platform, sag/no-sag correction)

ii) separately, the Operators Wellbore Survey Group (OWSG) have produced a consistent set of PUMs which cover most situations encountered in borehole surveying. This is a suggested set of PUMs and is not mandated in any way. It is up to users to decide whether it is appropriate for their needs. The OWSG set of models includes the ISCWSA generic MWD models in i)

It must be stressed that ISCWSA does not certify, verify or mandate the use of any PUM or survey tool.
3.1 Overview of the Error Model

The basic measurements which constitute a borehole survey generally consist of a number of measured depth, inclination and azimuth values, taken at discrete intervals along the wellpath.

Directional software will use these measurements and assumptions about the shape of the wellpath between the stations (typically minimum curvature algorithms) to determine the 3D position of the well as Northings, Easting, TVD co-ordinates.

The purpose of the error model is to evaluate the effects of the various physical factors which lead to errors in the survey measurements and hence to determine uncertainty in the 3D position.

For a given survey tool, a number of different physical characteristics will be identified which could lead to errors. The effect of each of these on the measured depth, inclination or azimuth at a particular survey station is evaluated and in turn the effect on the wellbore position is determined. The effect of each error is then accumulated along the wellpath and the contribution of all the individual errors are combined to give the final uncertainty in wellbore position.

Within the error model, this uncertainty is held as a covariance matrix which describes the uncertainty along each co-ordinate axis and the correlations between these uncertainties. In directional software this covariance matrix is commonly used to determine an uncertainty ellipsoid at a particular confidence level. This ellipsoid may be shown graphically, represented in reports or projected onto a given plane, in which case it becomes an ellipse and ellipse semi-major, semi-minor axes can be reported in the plane. The ellipsoids from neighbouring well paths are used in anti-collision calculations to determine whether drilling a well at that location is allowable or not.

For example, assume that we have a certain survey tool (either a gyro or MWD) which contains three accelerometers used to determine inclination. We consider that after calibration each sensor could exhibit a bias (or offset) error, which is a common way to consider sensor errors. From sets of test data across different tools and runs we determine the typical range of that bias error and quantify it as a standard deviation.

Then, for a given wellbore survey, we evaluate the effect that an x-axis accelerometer bias error, with that standard deviation, would have on the inclination and azimuth measurements which we obtained at each survey station. Note- measured depth in this example comes purely from wireline or drill-pipe measurements and accelerometer bias errors do not affect the depth readings.

By this procedure, the survey measurement uncertainty (the x-accelerometer bias error) has been converted into an associated angular uncertainty. From this we can determine the uncertainty in the 3D position of the well at each point along the survey run due to possible x-accelerometer biases.

We can repeat the same process for a y-accelerometer bias, for a z-accelerometer bias and so on for all the significant sources of error that we can identify for this tool. All of these error values are then accumulated to determine the position covariance matrix at each station along the well.
It should be noted that we are not evaluating the actual accelerometer bias values during this run. Instead, we are assessing the uncertainty in well position, due to the likely range of errors that we can anticipate for these sensors. The output answer is therefore a statistical estimate of the expected uncertainty for a particular survey.

In the description above, the uncertainties are repeatedly said to be ‘accumulated’ along the well. That accumulation happens on a statistical basis.

The errors caused by a specific error source (for example, the x-accelerometer bias error) from survey to survey may be correlated if the underlying sensor error value does not change. Other errors (such as pipe stand-up) may randomise from survey to survey and are said to be uncorrelated.

Where the errors are correlated (i.e. expected to have the same value from point to point) the uncertainties are added in the usual arithmetic way. However, if the errors are uncorrelated then we consider that they will be different from point to point and there is chance that different errors may cancel. In that case, since it is the standard deviation of the errors that we are dealing with, the uncertainties are root summed squared together.

When combining the contributions due to all of the individual error sources, it is a basic assumption of the model, that all of the individual error sources are independent (uncorrelated) from each other. This means that for example the actual x-accelerometer error of one measurement is independent from the y-accelerometer error at the same (or any other) survey station, as well as independent from the z-accelerometer error, the depth error, the sag error, etc. This independency allows for individual conversion into position uncertainties before summation.

Having described the model, we can now identify the various components that are required to run the calculations:

i) for a particular survey tool we have a number of error sources which effect downhole surveys. These are identifiable physical phenomena which will lead to an error in the final wellbore position; for example the residual sensor error after calibration.

ii) each error source has an error magnitude, which is the standard deviation of that error as determined from test data.

iii) each error source has a set of weighting functions, which are the equations which describe how the error source effects the survey measurements of measured depth, inclination and azimuth.

iv) each error source also has a propagation mode which defines how it is correlated from survey station to survey station, survey leg to leg and well to well, and this is used in accumulating the errors.

Typically these components are defined within the PUM for a particular tool and although not strictly necessary within the PUM, each error source generally has an associated:

v) error code string such as ABZ or MSZ. This is simply a shorthand identifier.
The mathematical framework of the error model includes the definitions of a wide range of error sources used to model MWD, gyro and utility tools. For each source, weighting functions are defined. A number of possible propagation modes are also defined.

The PUM for a particular tool will define which error sources required to model that tool, along with the appropriate magnitudes and propagation modes. Weighting functions may also be included in the PUM or may be inferred from the source identifier.

Before we discuss each of these items in detail, here is an example of how the error model works which should help to illustrate what these terms mean.
**Example 1: Declination error**

Downhole MWD tools measure magnetic azimuth and in order to calculate the true (or grid) north azimuth values, the declination term has to be added to the downhole data:

\[ A_t = A_m + \delta \]  

(1)

Usually, declination is determined from a global magnetic model like the BGGM or IGRF models. However, these work on a macro scale and may not be totally accurate in an oil field. So there is some uncertainty (or error bounds) on the declination value and this is clearly a possible source of survey error.

If we include a term \( \epsilon_{\text{dec}} \) for these errors then our above equation becomes:

\[ A_t = A_m + (\delta + \epsilon_{\text{dec}}) \]  

(2)

Therefore, the MWD model identifies an *error source* with the mnemonic *code* DEC which can be used to model declination uncertainty. From the above equation we can see that a declination error will lead directly to an error in the true azimuth, but it has no effect on inclination or depth measurements.

Hence the DEC *weighting functions* are \([0,0,1]\) (i.e. \(\text{md}=0, \text{inc}=0, \text{az}=1\)). These are about the simplest weighting functions you can have.

The standard MWD model gives the DEC error source a *magnitude* of 0.36°. If an In-Field Reference survey was carried out in the field then the declination uncertainty would be smaller and there could be a different tool model (PUM) for MWD+IFR with a smaller magnitude for this error source.

If we assume that, whatever the value, the declination is constant over the whole oil field then all MWD surveys, with all different survey tools and in all BHA used in all the wells in the field will be subject to the same error. Hence then DEC term has a global *propagation mode*.

Declination error is a function of the Earth’s magnetic field and has no influence on gyro survey tools, so the gyro model doesn’t need to include a declination error term.
3.2 Assumptions and Limitations of the Model

The ISCWSA error model is designed to be a practical method that can be relatively easily implemented in software and then used by well planners and directional drillers. It is intended to be applied to a range of tools, used worldwide and accordingly attempts to give good representative survey uncertainties without the need to model every single variation of tool or running conditions.

The model only applies to surveys run under normal industry best-practise procedures which include:

i. rigorous and regular tool calibration,
ii. a sufficiently short survey interval to correctly describe the wellbore
iii. field QC checks, such as total magnetic field, gyro drifts, total gravity field and magnetic dip angle on each survey measurement,
iv. the use of non-magnetic spacing for MWD surveys according to industry norms,

ISCWSA has produced a series of paper which describe the necessary QC process in more detail [10-11]

It should be recognised that the model cannot cover all eventualities and works on a statistical basis and so says nothing specific about any individual survey. The results can be interpreted as meaning that if a well was properly surveyed a number of times by a variety of different tools with the same specification, then the results would be expected to be randomly distributed with a range of values corresponding to the error model uncertainty results.

The model cannot cover gross blunder errors such as user error in referencing gyros, defective tools or finger trouble entering surveys into a database.

The model does not cover all variations and all possibilities in borehole surveying, for example survey data resolution is not currently modelled.

The model assumes that the wellbore can be adequately described by a constant arc between survey stations and it aims to evaluate how much errors in these measurements contribute to position uncertainty. No allowance is made for the survey measurements not being sufficient to define the wellpath. i.e. the model assumes that if we could take perfectly accurate inclination, azimuth and depth measurements we would have an exact value for the wellbore position. As a rule of thumb this is taken to be a survey interval of 100ft.

Finally, a major misconception is that the ISCWSA provides certified error models for specific survey tools. The published ISCWSA papers only define the process and equations to work from a set of error model parameters to an estimate of position uncertainty. The ISCWSA committee does not define, approve or certify the tool codes containing the actual error model magnitudes which drive the error model. These should be obtained from the survey contractor who provides the tool, since they are the ones best placed to understand the specifications and limitations of their tools.

The only exception to this is that there exists a generic ISCWSA MWD model which comes in eight variants for the combinations of with/without sag correction, with/without axial magnetic correction.
and from a fixed or floating platform. The Operators Wellbore Survey Group (OWSG) provide a more complete set of error models which are available for use.
4 Details of the Mathematical Framework

4.1 Definition of Axes

For clarity the following axes sets are used in the error model:

**Body Reference Frame (tool axes)**
The z-axis is coincident with the along hole axis of the survey tool and the x and y-axes are perpendicular to z and to each other. This is axes set used to describe orientations of the various sensors.

**Earth Centred Reference Frame (nev)**
The x-axis is in the horizontal plane and points toward true north, the y-axis is also in the horizontal plane and points towards true east. The z-axis points downwards.

**Borehole Reference Frame (hla)**
The z-axis is aligned along the borehole axis. The x-axis is perpendicular to z and points toward the high side. The y-axis perpendicular to both of these and hence is laterally aligned across the borehole.
4.2 Notation Used in the Mathematical Framework

Subscripts:
In the following discussion we will have need to identify and index the differences between different error sources, survey stations and survey legs. The following conventions are used throughout:

- $i$ used to index different error sources from 1...$I$
- $k$ used to index different survey stations in a survey leg, from 1...$K$
- $l$ used to index different survey legs in a well, from 1...$L$

The following terms are used in the error model framework:

- $\sigma_i$: the magnitude of the $i$th error source
- $3x1$ vectors:
  - $e_{i,l,k}$: the error due to the $i$th error source at the $k$th survey station in the $l$th survey leg
  - $e_{i,l,k}^*$: the error due to the $i$th error source at the $k$th survey stations in the $l$th survey leg, where $k$ is the last survey of interest
  - $\alpha p$: weighting function – the effect of the $i$th error source on the survey measurement vector
  - $\Delta r$: borehole displacement between successive survey stations

- $3x3$ matrices:
  - $d r$: the effect on the borehole positions of changes in the survey measurement vector
  - $[C]_{nev}$: error covariance matrix in $nev$-axes
  - $[T]_{hla}^{nev}$: $nev$ to $hla$ transformation direction cosine matrix

So for example, $e_{i,l_1,k_1}$ refers to the position error vector, in the $nev$ frame, due to the $i$th error source, at survey station $k_1$ in the $l_2$ survey leg.
4.3 Notation Used in the Weighting Functions

The following variables are used in the weighting functions:

- $A_m$: magnetic azimuth
- $A_t$: true azimuth
- $B$: magnetic total field
- $B_H$: horizontal component of magnetic field
- $B_{x}, B_{y}, B_{z}$: Sensor magnetometer readings in the $x,y,z$ tool axes
- $c$: Running speed
- $D$: along-hole depth
- $ΔD$: difference along-hole depth between survey stations
- $G$: Earth’s gravity
- $G_{x}, G_{y}, G_{z}$: Sensor accelerometer readings in the $x,y,z$ tool axes
- $h$: value of weighting function (used in recursive equations)
- $I$: inclination
- $α$: toolface angle
- $Ω$: Earth’s rotation rate
- $ϕ$: latitude
- $Θ$: magnetic dip angle
- $γ$: $xy$-accelerometer cant angle
- $f$: noise reduction factor for initialisation of continuous surveys
- $k$: logical operator for accelerometer switching
- $v_d$: gyro drift
- $v_{rw}$: gyro random walk
- $w_{12}$: misalignment weighting term
- $w_{34}$: misalignment weighting term

4.3.1 Note on the use of Azimuth

In borehole surveying we typically make use of three north references – true north, grid north and magnetic north and therefore we have to deal with three different definitions of azimuth. Care must be taken when evaluating the error model to use the correct azimuth in the correct place.

Magnetic azimuth, $A_m$, is used throughout the MWD weighting functions (section 11.2), since by their nature MWD tools measure from magnetic north.

Similarly true azimuth, $A_t$, is used throughout the gyro weighting functions (section 11.3) since by their nature, gyro tools measure from true north.

As defined above, throughout this document the $nev$-axes north axis is aligned with true north and hence true azimuth is used in the partial derivatives of the well position with respect to survey measurements (equation 8 – 12) and for creating the direction cosine matrix to transform between the $nev$ and $hla$ axes (equation 30).

Some implementations use a $nev$-set aligned with grid north. These results can be obtained either by a rotation by the convergence angle or by using grid azimuth in the appropriate equations.
Even the main published error model SPE papers [1,2] differ in this regard since the MWD paper assumes true north and the gyro paper assumes grid north. This causes great confusion when comparing results. The validation dataset sets at www.copsegrove.com are all detailed assuming nev aligned with true north.
4.4 Evaluation of Position Uncertainty

Once we have identified the error sources that will affect our surveys and specified the range of values these error sources may take, we need a means of using that information to determine position error ellipses.

The survey measurements that are taken downhole are the inclination of the wellbore, the azimuth of the wellbore and the along-hole, measured depth at discrete points. From that information 3-d wellbore positions are calculated in the appropriate co-ordinate frame by making assumptions about the path of the well between these survey stations. This is most often done with minimum curvature algorithms, although other options such as balanced tangential are possible.

The propagation mathematics follows this trail from error source to survey measurements to position co-ordinates to determine the effect of each error source on the position uncertainty.

The core equation of the error evaluation is:

\[ e_i = \sigma_i \frac{dr}{dp} \frac{dp}{\partial \epsilon_i} \quad (3) \]

This is a simple chain rule application. We can break this equation down to examine the various constituent parts.

Firstly
\( \epsilon \) represents the error source (e.g. magnetometer calibration error could be an error source \( \epsilon_i \))

\( i \) is used to index which particular error source we are considering

\( \sigma_i \) is the magnitude of the uncertainty for the \( i \)th error source (i.e. a scalar value, e.g. 70nT)

\( \frac{dp}{\partial \epsilon_i} \) are the weighting functions for this source.

These are the partial derivatives of the survey measurements (depth, inclination and azimuth) with respect to that error source. \( \frac{dp}{\partial \epsilon_i} \) is a 3x1 vector with one term for each measurement, i.e.

\[ \frac{dp}{\partial \epsilon_i} = \left[ \frac{\partial D}{\partial \epsilon_i}, \frac{\partial I}{\partial \epsilon_i}, \frac{\partial A}{\partial \epsilon_i} \right] \quad (4) \]

Hence \( \sigma_i \frac{dp}{\partial \epsilon_i} \) is size of the effect of the \( i \)th error source on the survey measurements at that point.

\( e_i \) is the size of the position uncertainty error in nev-axes due to error source \( i \) at the current survey station (a 3x1 vector)

\( \frac{dr}{dp} \) is the effect of the survey errors in md, inc and az on the wellbore position in the NEV axis, (i.e. a 3x3 matrix)
\[
\frac{dr}{dp} = \begin{bmatrix}
\frac{dN}{dM} & \frac{dN}{dE} & \frac{dN}{dA_z} \\
\frac{dM}{dE} & \frac{dM}{dE} & \frac{dM}{dA_z} \\
\frac{dM}{dV} & \frac{dM}{dV} & \frac{dM}{dA_z}
\end{bmatrix}
\]

(5)

For example, our error source might be for x-axis magnetometer bias errors. The magnitude for this source is estimated to be 70nT.

For a particular station in the well, \(\sigma_i \frac{\partial A}{\partial \varepsilon_i}\) gives the azimuth measurement uncertainty in degrees due to that error source. \(\sigma_i \frac{dr \partial p}{\partial \varepsilon_i}\) is the position uncertainty at that station in metres (or feet).

We need to be able to calculate the \(\frac{dr}{dp}\) matrix. Wellbore positions are calculated using one of the standard methods such as minimum curvature or balanced tangential, so over an interval the \(\frac{dr}{dp}\) matrix depends on the surveys at either end of the interval.

If we write \(\Delta r_k\) for the displacement between survey station \(k-1\) and \(k\) and hence \(\Delta r_{k+1}\) for the displacement between stations \(k\) and \(k+1\), then we can split \(\frac{dr}{dp}\) in to the variation over the preceding and following survey intervals and re-write (3) as:

\[
e_{l,l,k} = \sigma_i \left( \frac{d\Delta r_k}{dp_k} + \frac{d\Delta r_{k+1}}{dp_k} \right) \frac{\partial p_k}{\partial \varepsilon_i}
\]

(6)

Where now:

- \(e_{l,l,k}\) is the error due to the \(l\)th error source at the \(k\)th survey station in the \(l\)th survey leg
- \(\frac{d\Delta r_k}{dp_k}\) is the effect of the errors in the survey measurements at station \(k\), on the position vector from survey station \(k-1\) to survey station \(k\) and similarly
- \(\frac{d\Delta r_{k+1}}{dp_k}\) is the effect of the errors in the survey measurements at station \(k\), on the position vector from survey station \(k\) to survey station \(k+1\)

Although minimum curvature is the preferred method for calculating the wellbore positions, it is simpler to use the balanced tangential method to determine \(\frac{d\Delta r_k}{dp_k}\) and there is no significant loss of accuracy in the uncertainty results.

The balanced tangential model gives us the following equation for the displacement between any two survey stations \(j-1\) and \(j\) in the nev-axes:

\[
\Delta r_j = \begin{bmatrix}
\Delta N \\
\Delta E \\
\Delta V
\end{bmatrix} = \frac{D_j - D_{j-1}}{2} \begin{bmatrix}
sin l_{j-1} \cos A_{j-1} + \sin l_j \cos A_j \\
\sin l_{j-1} \sin A_{j-1} + \sin l_j \sin A_j \\
\cos l_{j-1} + \cos l_j
\end{bmatrix}
\]

(7)

So for the interval between stations \(k-1\) and \(k\) we can write:

\[
\frac{d\Delta r_k}{dp_k} = \begin{bmatrix}
\frac{d\Delta r_k}{dD_k} & \frac{d\Delta r_k}{dl_k} & \frac{d\Delta r_k}{dA_k}
\end{bmatrix}
\]

(8)
Substituting $j=k$ and differentiating equation (7) we get:

$$
\frac{d\Delta r_k}{dD_k} = \frac{1}{2} \left[ \frac{\sin l_k \cos A_{k-1} + \sin l_k \cos A_k}{\sin l_k \sin A_{k-1} + \sin l_k \sin A_k} \right]
$$

Putting these together:

$$
\frac{d\Delta r_k}{dp_k} = \frac{1}{2} \left[ \frac{\sin l_k \cos A_{k-1} + \sin l_k \cos A_k}{\sin l_k \sin A_{k-1} + \sin l_k \sin A_k} \right]
$$

Similarly, for the interval between stations $k$ and $k+1$ we can write:

$$
\frac{d\Delta r_{k+1}}{dp_k} = \left[ \frac{d\Delta r_{k+1}}{dD_k} \frac{d\Delta r_{k+1}}{dI_k} \frac{d\Delta r_{k+1}}{dA_k} \right]
$$

Substituting $j=k+1$ and again differentiating equation (7) we get:

$$
\frac{d\Delta r_{k+1}}{dD_k} = \frac{1}{2} \left[ \frac{-\sin l_k \cos A_k - \sin l_{k+1} \cos A_{k+1}}{-\sin l_k \sin A_k - \sin l_{k+1} \sin A_{k+1}} \right]
$$

And so

$$
\frac{d\Delta r_{k+1}}{dp_k} = \frac{1}{2} \left[ \frac{-\sin l_k \cos A_k - \sin l_{k+1} \cos A_{k+1}}{-\sin l_k \sin A_k - \sin l_{k+1} \sin A_{k+1}} \right]
$$

In summary, we have now calculated the 3x3 matrix equations which describe the uncertainty in the wellbore position, caused by errors in the survey measurement at any preceding given station, $k$. The 3x3 matrices are evaluated in the nev co-ordinate frame.

Since the wellpaths are built up as a number of curved sections, each of which depends on the attitude at either end, along most of the wellpath each survey measurement affects both the
interval which precedes it and the interval of wellpath that follows. However, at the last survey station of interest, only the preceding interval is applicable and equation (6) reduces to:

\[ e_{i,K}^* = \sigma_{iL} \left( \frac{d\Delta r_K}{d p_K} \right) \frac{dp_K}{d e_i} \]

(14)

Where superscript * indicates we are only considering the preceding interval and the use of capital K and L indicates we are considering the last station in the evaluation to that point.

Errors at the purple survey affect both intervals 1 and 2. Similarly the red survey affects intervals 2 and 3. But errors in the green survey will only affect interval 3. So the error at the green survey is

\[ \sum_{error\ sources} e_{purple} + e_{red} + e_{green}^* \]

Then when we take another survey, the error at the end of the well now has to include the effects of both the blue and green surveys on interval four i.e.

\[ \sum_{error\ sources} e_{purple} + e_{red} + e_{green} + e_{blue}^* \]
4.5 Derivation of Weighting Functions

The ISCWSA MWD and gyro models identify a range of error sources (currently 81) which contribute to errors in surveys from these tools. Each source has an associated set of three weighting functions which define how that error source affects the measured depth, inclination and azimuth measurements.

A complete list of current weighting functions are given in the Appendix and are also defined in the accompanying spreadsheet ListOfISCWSAWeightingFunctions.xlsx.

We will not detail the derivation of each of these weighting functions here. Instead we give a summary of the derivation and detail of one particular example.

The surveyed inclination and azimuths are obtained from the tool’s raw sensor measurements via certain survey equations. For example, a standard MWD tool will record three accelerometer and three magnetometer measurements $G_x, G_y, G_z, B_x, B_y, B_z$. The inclination and azimuth at each station are determined from the following equations:

\[
I = \cos^{-1}\left(\frac{G_z}{\sqrt{G_x^2 + G_y^2 + G_z^2}}\right) \tag{15}
\]

\[
A_t = \tan^{-1}\left(\frac{(G_xB_y - G_yB_x)\sqrt{G_x^2 + G_y^2 + G_z^2}}{B_z(G_x^2 + G_y^2) - G_z(G_xB_x - G_yB_y)}\right) + \delta \tag{16}
\]

Similar (but different) survey expressions exist for gyros tools, although the actual equations will depend on the tool sensor configuration. Similarly, these MWD equations have a different form if axial interference corrections are made.

The weighting functions can be derived from these equations by taking the partial derivatives of the survey equations with respect to the error source.

As an example, for a z-accelerometer bias error we require the partial derivatives of these equations with respect to the z-accelerometer sensor reading, $G_z$.

Instead of reading the correct value of $G_z^{true}$ the tool will actual give:

\[
G_z = (1 + \epsilon_G^{scale\text{factor}})G_z^{true} + \epsilon_G^{bias} \tag{17}
\]

where $\epsilon_G^{bias}$ and $\epsilon_G^{scale\text{factor}}$ represent the residual errors of the survey tool after calibration. This equation represents a fairly standard, first-order method for modelling the output of a sensor (almost any type of sensor), which it is known will not give perfect output.

The MWD model has an error source, coded ABZ for z-accelerometer bias errors which corresponds to this $\epsilon_G^{bias}$ term.
From equation (17) we can see that \( \frac{\partial G_z}{\partial \epsilon_{Gz}} \), the partial derivative of the Gz measurement with respect to the \( \epsilon_{Gz}^{bias} \) is 1.

From the MWD survey equations (15) and (16) above, we can see that the G\(_z\) term appears in both the inclination and azimuth equations (note, that the accelerometer readings don’t have any effect on measured depth and so the depth weighting function is 0). So the inclination and azimuth **weighting functions** are determined by taking the partial derivatives of these survey equations with respect to G\(_z\).

i.e. for ABZ the weighting functions are:

\[
\begin{align*}
\left[ \frac{\partial D}{\partial G_z}, \frac{\partial I}{\partial G_z}, \frac{\partial A_{true}}{\partial G_z} \right] \frac{\partial G_z}{\partial \epsilon_{Gz}} \\
\begin{bmatrix}
\sin I \cdot \tan \theta \cdot \sin I \cdot \sin A_m \\
0, G, G
\end{bmatrix}
\end{align*}
\]

Gyro tools can be designed a little differently – some systems also have a cluster of three accelerometers and the inclination weighting function will be the same as the MWD case (this is the gyro AXYZ-ZB term). Other gyro tools only have x and y accelerometers and use the assumed total gravity value, and therefore these tools would be modelled without a z-accelerometer bias term. We can see that the error sources which are included in any particular survey tool model depend on the design of that tool.

### 4.6 Singular Weighting Functions

For certain sources, the weighting functions, \( \frac{\partial p}{\partial \epsilon_i} \) are singular when the well is vertical. However the position uncertainty vectors are still well defined. In these cases, we go straight to the evaluation of \( e_{i,l,k} \) and \( e'_{i,l,k} \) via the equations:

For these cases,

\[
e_{i,l,k} = \sigma_{i,l} \frac{(D_{k+1}-D_{k-1})}{2} \begin{bmatrix}
VertWftFn_North \\
VertWftFn_East \\
VertWftFn_Vertical
\end{bmatrix}
\]

And

\[
e'_{i,l,k} = \sigma_{i,l} \frac{(D_K - D_{K-1})}{2} \begin{bmatrix}
VertWftFn_North \\
VertWftFn_East \\
VertWftFn_Vertical
\end{bmatrix}
\]

These replacements for the weighting functions in the North, East and Vertical axes are given in the Appendix.
4.7 Summation of Uncertainty Terms and Propagation Modes

The tool model for any particular survey instrument will include a number of different error sources, and we must consider all survey legs in the well and all the survey stations in each leg. So for a well we must add the error contributions over:

i. all survey legs in the well (index by \( l \))
ii. each survey station in each leg (indexed by \( k \))
iii. the contributions from each error source (indexed by \( i \))

Once we have calculated the contribution to the error ellipse from each error source, at each survey station in each leg of our well, we have to add up all these contributions. However, when doing this we have to take into account how the errors relate to each other at station and hence how the uncertainty values should be accumulated.

The basic form of the summation equation is:

\[
[C_k]_{\text{nev}} = \sum_{\text{errors}} \sum_{k_1 \leq K} \sum_{k_2 \leq K} \rho(e_{i,l_1,k_1}, e_{i,l_2,k_2}) \cdot e_{i,l_1,k_1} \cdot e_{i,l_2,k_2}^T \tag{22}
\]

Where \( \rho(e_{i,l_1,k_1}, e_{i,l_2,k_2}) \) is the correlation coefficient for the \( i \)th error source, between the results at the \( k_1 \) survey station in leg \( l_1 \) and the \( k_2 \) survey station in leg \( l_2 \).

The output we obtain is expressed in the form of a covariance matrix – a 3x3 matrix, in the \( \text{nev} \) axes, which describes the position uncertainty in each axis down the main diagonal and the correlations between these values in the off-diagonal terms.

In principle the correlations could have any value between -1 and 1, including zero for uncorrelated terms and also non-integer values. In practice however, the majority of the errors in borehole survey are either uncorrelated (\( p=0 \)) or fully correlated (\( p=1 \)) between different survey stations.

This means there are two basic cases:

1) The errors between survey stations are said to be correlated if they are directly linked and would have the same underlying error value from station to station.

So for example for a \( z \)-axis accelerometer bias error, since we using the same tool throughout a survey leg, we would expect this bias to have the same value from survey station to survey station. Hence the effects of the error will build all the way down the wellbore.

In which case, in one dimension, the uncertainty contributions are added in the usual arithmetic way:

\[
e_{\text{total}} = e_1 + e_2 \tag{23}
\]
2) If the errors are not linked from station to station then they are uncorrelated or statistically independent, e.g. if we have two independent error sources, then they could both cause a positive inclination error and add together but it is also possible that one might create a positive inclination error and the other a negative error.

In which case we are taking a random value from pot 1 and a random value from pot 2 and the error contributions must be root sum squared (RSS) together:

\[ e_{\text{total}} = \sqrt{e_1^2 + e_2^2} \]  

(24)

It is a basic assumption of the model framework that the statistics of the various different error sources are independent so they will be RSS’ed together – for example, there is no reason why sag error would be connected to z-axis magnetometer bias or to declination error etc. [This is the reason why \( e_I \) is not split into \( e_{I1} \) and \( e_{I2} \) in Eq. 21; the correlation \( \rho(e_{I1}, e_{I2}) \) is by assumption always 0 between different sources i1 and i2.]

Although the different error sources are independent from each other an individual error source may or may not be statistical correlated from survey to survey along the well.

The possible correlation between measurements depends as much on the tool configuration and measurement mode, as on the error source itself. For example, the z-axis magnetometer bias may be persistent for a particular surveying tool, and hence give correlated readings throughout a survey leg. However, if we go to another leg, using a different tool, the effect of this bias should not be correlated between the two legs. Similarly, an error source may behave correlated between survey legs in the same well, but independent between survey legs in different wells. The “lowest degree” of correlation occurs when any two measurements are independent, in which case the error source is termed random.

So a given error source may be independent at all surveys stations, or correlated between survey station- either just the stations within a leg, or over all legs within a well or over all wells within a field.

Therefore the model defines four propagation modes for the errors:

<table>
<thead>
<tr>
<th>Propagation Mode</th>
<th>Identifier</th>
<th>( \rho_1 )</th>
<th>( \rho_2 )</th>
<th>( \rho_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>R</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>always independent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Systematic</td>
<td>S</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>correlated from survey station to survey station</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Well by Well</td>
<td>W</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>correlated from leg to leg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global</td>
<td>G</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>correlated over all wells</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Where the separate correlation coefficients \( \rho_1, \rho_2, \rho_3 \) are defined as:

\( \rho_1 \) is the correlation between survey stations within the same survey leg

\( \rho_2 \) is the correlation between survey stations in different legs in the same well

\( \rho_3 \) is the correlation between survey stations within different wells in the same field
The propagation mode is a property of the error source and is defined in the tool model. In practice, most error sources are systematic within a leg or are random and only a limited number of well by well or global sources have been identified.

Reverting to our general summation equation (22) we can break down the overall summation of random, systematic and global/well by well error sources into:

\[
[C]_K = \sum_{i \in R} [C]_{i,K}^{rand} + \sum_{i \in S} [C]_{i,K}^{syst} + \sum_{i \in \{W,G\}} [C]_{i,K}^{well}
\] (25)

Then by applying the correlation coefficients above we can determine that the contribution of the random errors is given by:

\[
[C]_{i,l}^{rand} = \sum_{i=1}^{L-1} [C]_{i,l}^{rand} + \sum_{k=1}^{K-1} (e_{i,L,k})^T \left( \sum_{k=1}^{K} (e_{i,L,k}) \right) + (e_{i,L,k})^T (e_{i,L,k})^T
\] (26)

The systematic errors are:

\[
[C]_{i,l}^{syst} = \sum_{i=1}^{L-1} [C]_{i,l}^{syst} + \sum_{k=1}^{K} (e_{i,L,k})^T \left( \sum_{k=1}^{K} (e_{i,L,k}) \right) + (e_{i,L,k})^T (e_{i,L,k})^T
\] (27)

And finally the well by well and global errors:

\[
[C]_{i,L}^{well} = E_{i,L} E_{i,L}^T
\]

\[
E_{i,L}^{well} = \sum_{i=1}^{L-1} \left( \sum_{k=1}^{K} e_{i,L,k} \right) + \sum_{k=1}^{K} e_{i,L,k} + e_{i,L,k}
\] (28)

The individual terms for the various groups of error sources are given below. In these equations:
**Definition of ISCWSA Error Model Rev4**

\( e_{i,l,k} \) is the vector contribution of the \( i \)th error source, in the \( i \)th survey leg at the \( k \)th survey station (3x1 vector)

\( e^*_{i,l,k} \) is the vector contribution of the \( i \)th error source, in the \( i \)th survey leg at the last survey point of interest i.e. the \( K \)th survey station (3x1 vector)

\( i \) is the summation over error sources from 1...\( I \)

\( k \) is the summation of survey stations from 1...\( K \): the current survey station

\( l \) is the summation over survey legs from 1...\( L \): the current survey leg

The mathematical details of this process can be found in Appendix A.

The final output of the summation is a 3x3 covariance matrix, which describes the error ellipse at a particular station. In the \( nev \)-axes, the covariance matrix is:

\[
[C]_{nev} = \begin{bmatrix}
\sigma_N^2 & \text{Cov}(N,E) & \text{Cov}(N,V) \\
\text{Cov}(N,E) & \sigma_E^2 & \text{Cov}(E,V) \\
\text{Cov}(N,V) & \text{Cov}(E,V) & \sigma_V^2
\end{bmatrix}
\]  

(29)

Here \( \sigma_N^2 \) is the variance in the north-axis and the uncertainty in north axis (at 1-standard deviation) is \( \pm \sqrt{\sigma_N^2} \).

In the same way, the other terms on the lead diagonal are uncertainties along the other principle axes. The \( \text{Cov}(N,E) \), \( \text{Cov}(N,V) \) and \( \text{Cov}(E,V) \) terms are the covariances and give the skew or rotation of the ellipse with respect to the principle axes.

### 4.8 Transformation to Borehole Axes

The covariance matrix above is expressed in the earth-centred \( nev \)-axes, this can be transformed to the borehole reference frame, \( hla \) by pre- and post-multiplying the covariance matrix with the \( nev \)-to-\( hla \) direction cosine matrix, \( [T]_{nev}^{hla} \).

\[
[C]_{hla} = [T]_{nev}^{hla} [C]_{nev} [T]_{hla}^{nev}
\]

(30)

The direction cosine matrix can be obtained a rotation in the horizontal place to the borehole azimuth, followed by a rotation in the vertical to the borehole inclination and is given by:

\[
[T]_{hla}^{nev} = \begin{bmatrix}
\cos l \cos A & -\sin A & \sin l \cos A \\
\cos l \sin A & \cos A & \sin l \sin A \\
-\sin l & 0 & \cos l
\end{bmatrix}
\]

(31)
4.9 Position Uncertainty Model for a Specific Tool

The elements required to model a specific survey tool are:

1) The error sources which are defined for the tool (generally each source has an identifier, although this is not strictly essential).
2) The magnitude for each error source.
3) The units for that magnitude.
4) The propagation mode for each source.
5) The weighting functions to be invoked for depth, inclination and azimuth errors (either specified as formulae or by reference).
6) Optionally, the inclination range over which that source is to be applied.
7) Optionally, design parameters to be used in the evaluation (e.g. running speed or cant angle for certain gyro tools).

These items are often grouped together in what can be referred to as either a Position Uncertainty Model (PUM), Instrument Performance Model (IPM), tool code, IPM file or error model.

The PUM will generally have a name to identify which survey tool is models and may also include metadata such as revision number, comments on usage and applicability, and audit history (originator, source, status, tool type etc.)

Since most tool codes can be created in fixed or floating platform versions, with varying depth source magnitudes, some software now includes both sets of terms in the PUM and allows the software to select the correct depth terms to apply, depending how the site for that well is setup. Users should be aware of this complication if copying PUMs, since using all depth terms will result in errors.
5 Error Sources and Weighting Functions

5.1 Common Elements of Modelling

Although many of the components of the MWD and gyro models are necessarily quite different, the error sources which model depth and misalignment of the tool are the same. By this we mean that the mathematical formulae are the same, but obviously actually magnitudes in the PUM will depend on how a depth is obtained (e.g. wireline or pipe tally) and how a tool is centralised.

5.1.1 Depth Terms

Depth is covered with a reference term (which may be random or systematic), a scale and a stretch term. Depth errors are discussed in further detail in [3].

The same weighting functions are used for gyro depth errors. In general most PUMs can be created in two basic variants to cover the cases of surveys from a fixed rig (e.g. land rig) and from a floating platform. The depth reference terms vary between these cases.

For example, the generic MWD models use the following values for modelling drill-pipe depth in these two scenarios:

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Propagation Mode</th>
<th>Units</th>
<th>Fixed</th>
<th>Floating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth: Depth Reference – Random</td>
<td>DREF</td>
<td>R</td>
<td>m</td>
<td>2.2</td>
</tr>
<tr>
<td>Depth: Depth Reference – Systematic</td>
<td>DREF</td>
<td>S</td>
<td>m</td>
<td>0.35</td>
</tr>
<tr>
<td>Depth: Depth Scale Factor – Systematic</td>
<td>DREF</td>
<td>S</td>
<td>-</td>
<td>0.00056</td>
</tr>
<tr>
<td>Depth: Depth Stretch – Global</td>
<td>DST</td>
<td>G</td>
<td>1/m</td>
<td>2.5E-07</td>
</tr>
</tbody>
</table>

5.1.2 Borehole Misalignments

Borehole misalignments are handled in the same way in both the MWD and gyro models. This method avoids the complication of toolface dependency in the misalignments which was present in early versions and is considered to handle certain geometries, such helix-shaped, vertical boreholes better than the original MWD terms. There are four borehole error source terms and three possible calculation options which are handled via two weight parameters.

The full range of options is given by

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Depth</th>
<th>Inclination</th>
<th>Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>XYM1</td>
<td>0</td>
<td>$w_{12}$</td>
<td>0</td>
</tr>
<tr>
<td>XYM2</td>
<td>0</td>
<td>0</td>
<td>$w_{12}/\sin(I)$</td>
</tr>
<tr>
<td>XYM3</td>
<td>0</td>
<td>$w_{34} \cos(A_i)$</td>
<td>$- w_{34} \sin(A_i) / \sin(I)$</td>
</tr>
<tr>
<td>XYM4</td>
<td>0</td>
<td>$w_{34} \sin(A_i)$</td>
<td>$w_{34} \cos(A_i) / \sin(I)$</td>
</tr>
</tbody>
</table>
The calculation options are:

<table>
<thead>
<tr>
<th></th>
<th>$w_{12}$</th>
<th>$w_{34}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative 1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Alternative 2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Alternative 3</td>
<td>$\sin I$</td>
<td>$\cos I$</td>
</tr>
</tbody>
</table>

Alternatives 1 and 2 have their own strengths and weaknesses, whereas Alternative 3 is designed to combine the best of both options and is the preferred calculation option. This is discussed in detail in Appendix B of [2] and in [4].

Hence the current ISCWSA generic MWD models use alternative 3 and the weighting functions reduce to:

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Depth</th>
<th>Inclination</th>
<th>Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>XYM1</td>
<td>0</td>
<td>$\text{Abs} (\sin I)$</td>
<td>0</td>
</tr>
<tr>
<td>XYM2</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>XYM3</td>
<td>0</td>
<td>$\text{Abs} (\cos I) \cdot \cos (\text{Az}$</td>
<td>$-(\text{Abs} (\cos I) \cdot \sin (\text{Az}) / \sin I)$</td>
</tr>
<tr>
<td>XYM4</td>
<td>0</td>
<td>$\text{Abs} (\cos I) \cdot \sin (\text{Az})$</td>
<td>$(\text{Abs} (\cos I) \cdot \cos (\text{Az}) / \sin I)$</td>
</tr>
</tbody>
</table>

Note XYM3 and XYM4 are singular in vertical hole. The singular versions are given in Appendix A. XYM2 is also singular when vertical is misalignment option 1 is used. However as noted in the [2] in this situation this term may give strange/unwanted values when azimuth or toolface vary.

Borehole misalignments may be modelled as either random or systematic propagation, depending on whether the toolface of the survey tool is expected to vary. Systematic is the more conservative option is more generally implemented.

5.1.2.1 Sag

A separate term models the deflection of the BHA under gravity, which can result in the inclination readings from the survey tool not being aligned with the axis of the borehole. The MWD tool has a SAG term and in the gyro model this has at times been referred to as VSAG. These are the same error sources with the same weighting functions.
6 MWD Modelling

In general ISCWSA does not generate position uncertainty models for particular survey tools. The exception to this rule, are the models for generic MWD tools, for which does ISCWSA does define eight PUMs. These cover the various combinations of uncorrected and axial-corrected MWD, in fixed and floating platform variants, with and without sag correction. The error model maintenance committee does not create PUMs for any further MWD variants (such as IFR1, multi-station versions etc.)

I.e. the Error Model Maintenance Committee provides PUMs for the following cases.

<table>
<thead>
<tr>
<th>MWD_Fixed Platform</th>
<th>MWD+AxialCorr_Fixed Platform</th>
</tr>
</thead>
<tbody>
<tr>
<td>MWD_Floating Platform</td>
<td>MWD+AxialCorr_Floating Platform</td>
</tr>
<tr>
<td>MWD+Sag_Fixed Platform</td>
<td>MWD+AxialCorr+Sag_Fixed Platform</td>
</tr>
<tr>
<td>MWD+Sag_Floating Platform</td>
<td>MWD+AxialCorr+Sag_Floating Platform</td>
</tr>
</tbody>
</table>

6.1 MWD Revision 4 Position Uncertainty Models

The latest version of the MWD model is revision 4, agreed in June 2015. The ISCWSA Rev4 MWD models are identical to those issued by the OWSG in their Rev2 release.

The full details of the PUMs for this version, along with details of all previous MWD models can be found in Excel spreadsheet form on the ISCWSA website:


Revision 4 Validation Datasets

Test case results for this revision, on the three ISCWSA test profiles can be found on the Copsegrove website at www.copsegrve.com under OWSG Survey Tool Error Models \ OWSG Set A models.
6.2 Weighting Functions

The revision 4 model contains 34 weighting functions:

- 4 depth terms
- 4 borehole misalignment terms
- 1 sag term
- 20 terms for bias and scalefactor errors on the sensors
- 4 terms for reference field errors
- 1 drillstring interference term.

Details of all of the weighting functions can be found in the appendix to this document and also in the spreadsheet referenced above which details the PUMs.

6.2.1 Sensor Terms

The model includes bias and scalefactor errors for all the sensors in the tool. Since revision 3, these are modelled using toolface independent weighting functions following the methodology described in [13]. This combines the x and y axis sensor terms which end up being represented by two biases and three scalefactors for the accelerometers and a similar number for the magnetometers. The axial terms remain as a single bias and scalefactor for the z-accelerometer and z-magnetometer. This is a total of fourteen sensor terms.

The model covers both the variations of standard MWD and MWD with an axial magnetic interference correction (so called short-collar or single station corrections.). This results in a complete second set of sensor error terms. Only one set of sensor terms will be valid for any given situation.

For each sensor type, one of the scalefactor terms always propagates and systematic but the remainder may propagate as random or systematic depending on whether sliding or rotating drilling is modelled. In practise, the more conservative option of systematic propagation is generally used and that is what is quoted in the published ISCWSA PUMs.

Note that two of the cross axial accelerometer terms are singular in vertical hole, the modified version of the weighting functions are also given. Chad Hanak has produced a document which describes in the detail the derivation of the singular terms for the various versions of the error model [10].

6.2.2 Drillstring Interference

MWD users should model the expected magnetic interference from the BHA and hence determine suitable non-magnetic spacing distances. Revision 4 assumes that the BHA is spaced in this way to within a specified amount of magnetic interference in nT. There is now one term, AMIL which models drillstring interference. In the generic models this level is assumed to be 220nT at 1-sigma.
6.2.3 Geo-magnetic Reference

Two error sources are included for declination error (two sources one constant and one proportional to the horizontal component of the Earth’s field) and one each for total field and dip. The total field and dip terms are only used in axially corrected models.

The latest revisions include both systematic and random versions of all geo-magnetic terms. The random terms have relatively little impact on the ellipse sizes but are included for consistency and for use when deriving implied QA/QC limits.

The values used in the generic MWD models are those generally associated with standard definition magnetic models (such as the BGGM).

Values for other models such as the IGRF and high definition models have been suggested and may be used. These values are:

<table>
<thead>
<tr>
<th></th>
<th>MFI (nT)</th>
<th>MDI (Deg)</th>
<th>AZ (Deg)</th>
<th>DBH (Deg/nT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BGGM</td>
<td>130</td>
<td>0.20</td>
<td>0.36</td>
<td>5000</td>
</tr>
<tr>
<td>HDGM / MVHD</td>
<td>107</td>
<td>0.16</td>
<td>0.30</td>
<td>4118</td>
</tr>
<tr>
<td>IGRF / WMM</td>
<td>157</td>
<td>0.24</td>
<td>0.43</td>
<td>6029</td>
</tr>
</tbody>
</table>

6.2.3.1 Geomagnetic Lookup Tables

Modelling the uncertainty in worldwide geomagnetic reference terms with only four terms is clearly a simplification of a much more complex topic.

To provide greater detail the BGS have published various lookup tables for the accuracy of the BGGM model [8]. The lookup table may be used instead of the fixed term versions.

However, there is a complication with utilisation of the lookup tables. The mathematics of the error model is based on the manipulation of standard deviations, and no assumption is made about the distribution of errors. That is only required if one wishes to quantify probabilities. However, there is a general assumption by most users that the errors would be Gaussian.

As detailed in [6] it appears that the errors in the global geomagnetic models are in fact, non-Gaussian and can be best modelled with a Laplacian distribution. This has a greater likelihood of values in the tails of the distribution. This presents some problems in the implementation, especially when varying the number of standard deviations at which to report the position uncertainties.

The current recommendation is to define in advance the number of standard deviations required for output of the position uncertainty and read that across to a confidence level assuming Gaussian statistics. From that, determine the magnitude in the magnetic look up tables at that confidence level. Divide this value by the number of standard deviation required to get an ‘equivalent Gaussian
standard deviation’ (valid only at the confidence level in question) and then use that value as normal in the subsequent error model calculations. In that way, when the error model results are scaled back up to the required number of standard deviations, the geomagnetic terms will be reported at the correct confidence level.

That is, if reporting ellipses at 2 standard deviations (95.4% confidence in one dimensional Gaussian distributions), utilise the 95.4% look up table, read of the error source magnitudes at the given latitude and longitude of the well site. Divide those numbers by 2 to get the standard deviations to obtain an equivalent error source magnitude for use in the calculations.

Currently, the lookup tables are optional and are not considered as a revision of the model.

6.3 History of the MWD Error Model
There have been several revisions to the MWD error model over the years. Concise details of all the versions may be found in Excel spreadsheet form on the Error Model Management Group webpage as given in the previous section. This page will give a brief overview of the changes:

The revisions to the MWD Error Model are:

<table>
<thead>
<tr>
<th>Revision</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rev 0</td>
<td>As per SPE 67616 together with a number of typographical corrections [4]</td>
</tr>
<tr>
<td>Rev 1</td>
<td>Changed to the gyro style misalignment with 4 terms and calculation options [4]</td>
</tr>
<tr>
<td>Rev 2</td>
<td>Changes to the parameter values for the depth scale and stretch terms [4]</td>
</tr>
<tr>
<td>Rev 3</td>
<td>Replacement of all toolface dependant terms. [5]</td>
</tr>
<tr>
<td>Rev 4</td>
<td>Introduction of AMIL term and changes to misalignment magnitudes. Random magnetic reference values introduced to the main MWD model</td>
</tr>
</tbody>
</table>

6.3.1 Revision 0
The MWD error model was originally published as SPE 56702 in October 1999. This paper was updated and was published in SPE Drilling and Completion as SPE 67616 [1], in December 2000. The paper covers three distinct areas. It lays out the framework of the ISCWSA error model as discussed in the previous section, it defines the error sources applicable to MWD tools and it provides error magnitudes for these values, complete with a technical justification.

After, the publication of the SPE 67616, a small number of typographical errors identified and corrected and this defined as revision 0.

6.3.2 Revision 1
This revision changed how borehole misalignments were handled in the MWD model by adopting the same methodology as defined for the gyro model in [2]. The existing MX and MY misalignments were deprecated and replaced with the XYM1, XYM2, XYM3, XYM4 sources described above.
6.3.3 Revision 2
Revision 2 made changes to the various depth error magnitudes for both fixed and floating platforms. The consensus of the committee was that the previous depth terms were incorrect.

6.3.4 Revision 3
Rev3 replaced the 16 toolface dependant weighting functions with 20 new ones, following a method developed for the gyro error model. This removes the need to either include survey toolface, or to use methods to evaluate at the planning stage which toolfaces might be observed, a process which can give rise to unexpected results.

The new terms replace all the existing x and y accelerometer and x and y magnetometer bias and scalefactor terms, for both the standard MWD and MWD with axial correction cases. The suffix _TI1, _TI2 etc. is often used to differentiate these terms from the Rev2 sensor terms, where TI1 stands to Toolface Independent source 1 etc.

The new terms pull together the x and y effects, and the propagation mode varies from either random, where the toolface varies between survey stations and systematic for sliding between survey stations with constant toolface. In practise for MWD the random propagation would not normally be considered at the planning stage. The details of revision 3 are dealt with in [5].

6.3.5 Revision 4
Changes in revision 4:
1) The magnitudes of the borehole misalignment terms were increased from 0.06 deg to 0.1 deg. This change was implemented because, after consideration, the group felt that the existing values were too optimistic particularly in top hole. Hence ellipse sizes can be expected to be larger in top hole.

2) Replacement of the existing AMID and AMIC drillstring interference terms (which had units in degrees) with the AMIL term (which is specified in nano-Tesla). This reflects a change in how many companies do their non-mag spacing calculations. The older terms followed the philosophy in SPE67616, “A well-established industry practice is to require nonmagnetic spacing sufficient to keep the azimuth error below a fixed tolerance, typically ~0.5° at 1 s.d. for assumed pole strengths and a given hole direction. This tolerance may need to be compromised in the least favourable hole directions.” The use of the AMIL term assumes that BHA’s are designed with a specific length of non-mag and hence a consistent level of expected drillstring magnetic interference. The effect that this magnetic interference has on azimuth will then vary dependant on the well inclination, azimuth and the horizontal component of the Earth’s magnetic field. For the same BHA, large angular errors can be expected at higher latitudes. A magnitude of 220nT was chosen for AMIL, as a reasonable generic value. This gives reasonable agreement to the old model at mid-latitudes. However, the behaviour of the AMIL term is inherently different to AMIC+AMID and hence the error model will give different results depending on the well orientation and location.
3) Addition of DECR, DBHR, MDIR and MFIR terms to model random fluctuations in the geomagnetic reference field for declination, total field and dip. These terms were added for consistency with some of the commonly used IFR models. They will have a limited effect on the ellipse sizes, but will influence any field acceptance criteria derived from the error model values.

6.3.6 Bias Models
Early revision of the model included biased terms for depth and drill string interference terms.

It is well recognised that using drill pipe measurements on surface and the driller’s tally results in an underestimate of the true wellbore measured depth, since drill pipe will stretch due the suspended weight and will expand as temperature increases in hole. Similarly there has been some evidence that drill string interference terms are not completely random. Therefore bias terms were included in the model which had the effect of moving the centre of the survey ellipses away from the recorded survey station.

From revision 3 onwards ISCWSA advice was that bias model should not be used. They tend to confuse users and if the size of the bias error is significant for a survey application the recommendation would be to actually correct for the bias (with depth or interference corrections) rather than to move the ellipses.

Therefore from revision 3 bias terms have been deprecated.
7 Gyro Models

The core mathematics of the gyro model is designed to be independent of the technology used and it should be capable of modelling all systems currently in use or which have been foreseen. As before the specifics for a given tool/technology are contained in the specific error sources, magnitudes and propagation modes defined in the position uncertainty model for that tool.

Unlike MWD tools, which generally have a similar sensor configuration, gyro survey tools come in various different designs and can operate in two different ways. This means that although the basic ISCWSA framework is still used for modelling gyro tools the details of the models are more complicated and some additional features are needed.

The additional considerations are differing sets of weighting functions depending on the sensor configuration and two operating modes – stationary and continuous mode – with the model transitioning between these modes at defined inclinations.

In stationary (or gyro-compassing mode), the gyro is held at a fixed point in the borehole and the sensor readings as used to determine the inclination and azimuth of the tool relative to the Earth’s axis of rotation. An independent assessment of the inclination and azimuth is made at each survey station and depth may come from wireline or pipe tally.

In continuous mode the gyro is first initialised to define it’s inclination and azimuth and then the gyro sensors record changes to that initial orientation to attitude at subsequent points. Therefore all the later azimuths are dependent on the initial heading value. That initial azimuth may come from a stationary gyro-compass or made be defined by the user from an external reference source.

In such a case, a system may use stationary mode when near vertical and then switch over to continuous mode once the inclination builds above a set value. It is possible for the tool to move back to gyro-compassing mode if the hole angle drops again.

Stationary gyro mode is quite similar to the way in which MWD operates, and, aside from different weighting, functions the model behaves in a similar way. Weighting functions are evaluated at each station as a function of the depth, inclination, azimuth and some reference parameters (such as latitude). However, it should be noted that the weighting functions depend on TRUE azimuth and not MAGNETIC azimuth as in the MWD case.

Continuous mode is quite different as the weighting functions at each station are evaluated recursively i.e. they are dependent on their value at the previous survey station.
7.1 Sensor Configuration

Nearly all MWD tools consist of three orthogonal accelerometers and three orthogonal magnetometers.

However gyro tools can be built with various sensor configurations, with two or three accelerometers and with one, two or three gyros. The sensor configuration influences the navigation equations and hence there are different weighting functions in each case.

So, for the gyro model we have eight groupings of weighting functions i.e. for the stationary modes we have:

<table>
<thead>
<tr>
<th>Stationary mode</th>
<th>No. of Weighting Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor Configuration</td>
<td>Inclination</td>
</tr>
<tr>
<td>XY Accelerometers</td>
<td>4</td>
</tr>
<tr>
<td>XYZ Accelerometer</td>
<td>4</td>
</tr>
<tr>
<td>XY Gyro</td>
<td></td>
</tr>
<tr>
<td>XYZ Gyro</td>
<td></td>
</tr>
<tr>
<td>External Initialisation</td>
<td></td>
</tr>
</tbody>
</table>

As for the MWD model, the weighting functions are tool face independent. However the accelerometer terms are somewhat simplified in that their azimuth components have been ignored. So the accelerometer sources only have inclination weighting functions (depth and azimuth are zero). Similarly the gyro sources only have azimuth weighting functions.

For the gyro scalefactor terms, these axes have been lumped together by RSS-ing or by approximations. Both systematic and random versions of many of the terms are included.

The only additional complications are:

i. a cant angle and associated logical operator used in xy-accelerometer systems. This is for systems where the xy-accelerometer package is mounted (canted) at an angle to the body reference frame.

ii. a noise reduction factor which may apply to the gyro random noise in the stationary modes.

Both cant angle and the noise reduction factors are design parameter in the position uncertainty model for a given tool.

The logical operator is used to change the sign of the cant angle depending on the inclination of the tool i.e. k=1 when I ≤ 90° and k=-1 when I>90°. Some implementations achieve the same end in a more flexible manner by defining the cant angle in various range of inclinations.

Then for the continuous modes we have the following sensor configurations:

- XY Gyro
- Z Gyro
XYZ Gyro

each configuration has two weighting functions, one for a gyro drift term and the second a gyro random walk term.

Depending on the tool configuration the appropriate group of weighting functions would be invoked in each mode, although not all of the weighting functions in the group may be used. So a stationary tool PUM might have XYZ Accel and XY Gyro weighting functions. We would not expect to see weighting functions from both the XYZ Accel and XY Accel group in the same PUM. We do sometimes see XY Accel, XY Static, Z Continuous Gyro and XY Continuous Gyro all in the one PUM.

All the required weighting functions are listed in the Appendix. Derivations of the weighting functions can be found in [2].

ISCWSA does not define any gyro models for use.

7.2 Operating Modes

As outlined above, the gyro model considers two distinct operating modes for the tool.

Firstly, stationary mode is quite similar to the MWD model, where the weighting functions are evaluated at each survey station based purely on the current measured depth, inclination and azimuth and the physical details of the reference field – the total gravity value, the latitude and the constant of Earth’s rotation etc.

However, now a continuous mode is also introduced. In continuous mode, weighting functions are evaluated recursively i.e. the new value of a weighting function depends on the value it had at the previous survey station plus and additional increment.

In reality the sources that are important in continuous mode cause the attitude errors to build over time. In order to have a means of estimating elapsed time between surveys we evaluate the change in measured depth divided by the tool running speed. This running speed is another tool design parameter, which is defined in the position uncertainty model, along with the tool magnitudes.

Continuous mode weighting functions are written in the form:

\[ h_i = h_{i-1} + \frac{\Delta D_i}{c} \]  (32)

Where \( h_i \) is the new value of the weighting function, \( h_{i-1} \) is the value of this weighting function at the previous survey station, \( \Delta D_i \) is the change in depth between the two stations and the \( c \) is the running speed.
As a distance divided by speed, \( \frac{\Delta D_i}{c} \) has units of time. Evaluation of this term should be such that the time units match the biases and random walks (most often defined in deg/hr and deg/\( \sqrt{\text{hr}} \) respectively). Similarly, obvious the units for D and c must match (m and m/hr or ft and ft/hr).

The transition from stationary to continuous mode is considered to occur at a given inclination. So in the gyro model the sources now have a specified inclination range in which to be evaluated.

Before the transition inclination, the tool would operate in stationary mode and the weighting functions evaluated as normal. The weighting functions for the continuous sources are zero at this point.

After the transition the continuous weighting functions are evaluated. However, the azimuth uncertainty accumulated in the stationary sources is still required, since the subsequent azimuth measurements depend on the value at transition. Therefore either the stationary weighting functions are ‘frozen’ so they continue to give the same values throughout the continuous running, or the total azimuth error at transition can be moved into the \( \text{EXT-INIT} \) term. Note also that at transition any random gyro source which are frozen should change to systematic propagation for the remainder of that survey leg.

To put this in context, a given tool may be initialised in the stationary mode and transition to continuous mode once the hole inclination builds above 15°. All the surveys before the transition are stationary surveys, and the PUM will define the appropriate stationary functions to use. At transition to continuous mode the total azimuth uncertainty from all the stationary sources is 0.6°, via

\[
\sum_{i \in \text{stationary gyro sources}} d_{i,l} \frac{\partial A \zeta_K}{\partial \epsilon_i}
\]

After transition, the stationary sources are all frozen so that they continue to give an azimuth error of 0.6°. However, the error vector, \( \mathbf{e} \), due to these sources will of course change since although the outer terms are fixed, the geometry term in the brackets in equation (6) will vary at the subsequent stations.

\[
e_{l,i,k} = \sigma_{i,l} \left( \frac{d \Delta r_k}{dp_k} + \frac{d \Delta r_{k+1}}{dp_k} \right) \frac{\partial p_k}{\partial \epsilon_i}
\]

In addition to these stationary sources, the continuous terms will also come into play. The accelerometer terms, which determine the inclination uncertainty, are evaluated as stationary terms throughout.

If the tool were to drop back below 15°, the tool would drop back into stationary mode. In which case the model behaves in stationary mode exactly as before. The stationary weighting functions would be ‘unfrozen’ and evaluated as normal and the continuous weighting functions would be zero.
To stop unnecessary switching back and forward between modes (as might occur on a tangent hole section close to the transition inclination) the model includes provision for a minimum along hole distance between transitions. This is included in the test models but is rarely (if ever) seen in real models.

Another complex situation that is seen is a tool which is in stationary mode from 0° to 3° inclination, then transitions to z-gyro continuous mode until 15° before further transitioning to xy-gyro continuous mode. In this case the z-gyro weighting functions are frozen at 15°.
8 Utility Models

8.1 Inclination Only Surveys
ISCWSA’s position is that inclination only surveys do not constitute a true survey of wellbore and we recommend that they not be used. However, there are many legacy inclination only surveys in the industry and these should be handled in a suitable and consistent manner. There ISCWSA has produced a separate guidance document on the handling of inclination only surveys [11]. The main concept of this paper is that the wells will be considered to be vertical, but that uncertainty envelope will contain both the uncertainty due to the accuracy of the inclination measurements but also the uncertainty as to where the well is in space since the azimuth is not determined. This is achieved via use of the misalignment terms.

8.2 CNI and CNA
In addition to modelling of MWD and gyro tools, there is a need to cover other situations such as Blind Drilling or Unknown survey tools. ISCWSA does not provide these models although they are included in the OWSG set.

It is part of the design of the error model that the various terms correspond to recognisable and measurable physical sources of error. Clearly this cannot be the case of a blind drilling tool. Two additional weighting functions are commonly used in modelling these tools. These are CNI and CNA. Although their effect can be achieved via the misalignments, these terms serve a purpose to hold unattributed errors.
8.3 Testing and Validation

For validation of the error models, ISCWSA has defined three test wells. These are an extended North Sea well, a Gulf of Mexico fish hook well and a Bass Strait designer well.

Details of the test wells can be found in the accompanying spreadsheet ISCWSATestWells.xlsx and in reference [1].

Steve Grindrod has made diagnostic files available on the Copsegrove website (www.copsegrove.com) which may be used to validate an error model implementation. These files provide covariance values in both nev and hla axes for each error source at each survey station as well as total covariances.

Files are provided for:
   i. the latest OWSG models, which include the ISCWSA Rev4 MWD models.
   ii. Rev3 and Rev2 MWD models.
   iii. many of the gyro test cases defined in the gyro paper [2].

The format of these files and the definition of test wells is consistent throughout and they are the best resource available for implementers.

In addition the original test results are available in the main SPE papers [1,2].

The MWD paper provides test cases with the Revision 0 MWD models. These report uncertainty values (square root covariance terms) and correlations in the borehole axes at the end of the wells for MWD, MWD+Axial and MWD Bias models. These tests include an MWD and MWD+Axial tie-on.

The gyro paper [2] defines six test tool models and provides full covariance results at a number of depths stations. Covariances are reported in the NEV axes. It should be noted that the Copsgrove files and the MWD paper assume that the test wells were defined to true north. The gyro paper, however, assumes that the azimuths are to grid north. This will cause differences in the outputs.

Moreover some of the gyro test models change mode from stationary to continuous at an inclination which is not one of the defined survey stations in the well. Therefore the Copsegrove files add an additional survey station to ensure there is no ambiguity about where the mode transition occurs.

Note also that the models in the gyro paper were purely for testing of the software implementation. They are not to be used to model real world gyro tools.

ISCWSA does not define specific pass/fail standard for software testing. However, based on a statement in the gyro paper, results within ±1% (or ±2 units when the absolute covariance matrix value is less than 200) for total covariance matrix would generally be considered a correct implementation.


9 Implementation

9.1 Inputs
The inputs to the error model calculation are
1) the wellpath surveys – a list of measured depth, inclination and azimuth at each station.
2) position uncertainty model(s) for the tool(s) of interest in that well, defining the error sources, magnitudes, propagation modes and weighting functions for those sources.
3) a small number of reference quantities used in the weighting functions. These are:
   Total Magnetic Field
   Magnetic Dip Angle
   Acceleration due to gravity at the location
   Latitude

9.2 Output
The output at every survey station is a covariance matrix (3x3 symmetric matrix) in a given coordinate system (typically NEV or HLA). This gives the variance of the errors in each axis along the lead diagonal and the covariance of the errors in the off-diagonal terms.

9.3 Software Flow
A software implementation needs to loop through all the error sources in PUM, working down each leg and each survey in the well to evaluate the position error vectors.

\[ e_{i,l,k} = \sigma_{l,i} \left( \frac{d\Delta r_k}{dp_k} + \frac{d\Delta r_{k+1}}{dp_k} \right) \frac{dp_k}{p_i} \]  (6)

\[ e^{*}_{i,l,k} = \sigma_{l,i} \left( \frac{d\Delta r_K}{dp_K} \right) \frac{dp_K}{p_i} \]  (14)

Therefore the weighting functions and the geometric terms in the brackets need to be evaluated at each survey station.

Then, depending on the propagation modes defined in the PUM, the \( e \) and \( e^{*} \) vectors for each source need to be accumulated to the current survey station via equations (26) to (28). The contributions due to each source are then added as shown in (25) to define the total nev-covariance matrix for that station (29).

If required, the nev-covariance matrix may be transformed into the hla-axes.
The process then continues to the next survey station.

Below is a possible flowchart for evaluation of the error model results.
Surveys
Tool PU
Models
Wellpath
Reference
Data
Determine all error sources used over all survey legs
Calculate drdp matrices for all survey stations
Loop through all error sources, $i$
Loop through all survey legs, $l$
Initialise $e = e_{Star} = 0$

Loop

Loop

Loop

END

Store for output

START

Loop through all survey stations, $k$
Is well vertical and weighting function singular
NO
Evaluate weighting function $dpde$
Calculate $e$ and $e^*$
Depending on propagation mode update source NEV covariance and running sums. Remove $e^*\_Prev$ contribution and add in $e\_Prev$ and $e^*$
Transform NEV covariance to HLA if required.
Add this source covariance to sum of all sources at this station
Store for output

YES
Evaluate $e$ and $e^*$ directly using singular equations

Error Model Flowchart
9.4 Confidence Level
The error magnitudes defined in the MWD paper are all at one standard deviation (1-sigma). If the user requires final error ellipses at two standard deviations (or three) then the 1-sigma ellipses can simply be multiplied up and typically drilling software has a user control to define at what level error ellipses are to be output. There are two provisos to this – some drilling software packages allow the user to enter error magnitudes at different defined confidence levels (e.g. 1-sigma or 2-sigma etc.) Also, the optional look up tables for the geo-magnetic reference terms require the user to define the required output confidence level first so that certain magnetic field reference terms can be correctly calculated (for further details see section §6.2.3 above.)

9.5 Example Implementation
As an example a series of Excel spreadsheets accompany this document. These evaluate the Rev4 MWD model on the three test wells and provide comparison of the output values at certain survey stations with the diagnostic data at Copsegrove.com

All the errors are accumulated in covariance matrixes for each source and then these are accumulated for the entire well.

Similar sheets are available for some of the gyro test cases. As per the test data in the gyro paper, final covariance matrices in the nev axes (grid –nev) are calculated for a small number of survey stations in each well.

In order to get close agreement with the test values provided in the gyro paper, the wellpath is assumed to be defined to grid north, the curved sections of hole are defined at 10m interval and the Md=0 station in the well is assumed to be a survey station with associated errors
10 References

1. *Accuracy Prediction for Directional Measurement While Drilling*
   Hugh Williamson, SPE-67616

2. *Prediction of Wellbore Position Accuracy When Surveyed with Gyroscopic Tools*
   Torgeir Torkildsen, Stein Harvardsen, John Weston, Roger Ekseth, SPE-90408

3. *Quantification of Depth Accuracy*
   A.Brooks, H Wilson, A.Jamieson, D.McRobbie, S.G.Holehouse SPE-95611

4. *The Reliability Problem Related to Directional Survey Data*
   R.Ekseth, K.Kovalenko, J.L.Weston, T.Torkildsen, E.Nyrnes, A.Brooks and H.Wilson
   ADC/SPE 103734, IADC/SPE Asia Pacific Drilling Technology Conference and Exhibition,

5. *High-Integrity Wellbore Surveys: Methods for Eliminating Gross Errors*
   R.Ekseth; T.Torkildsen; A.Brooks; J.Weston; E.Nyrnes; H.Wilson and K.Kovalenko
   SPE/IADC 105558 SPE / IADC Drilling Conference, Amsterdam, The Netherlands, 20-22 (Feb 2007)

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   S. Grindrod, rev6 8/10/09 available at www.copesegrove.com

7. *MWD Toolface Independent Error Terms*
   S. Grindrod CDR-SM-03 Rev4 November 2009 available at www.copesegrove.com

8. *Confidence Limits Associated with Values of the Earth’s Magnetic Field used for Directional Drilling*
   Susan Macmillan, Allan McKay, Steve Grindrod SPE/IADC-119851

9. *An Introduction to Wellbore Positioning*
   A.Jamieson et al., published by University of the Highland & Islands

10. *Correct Vertical Well Weighting Functions for Cross-Axis Accelerometer Bias Terms*
    Chad Hanak, SuperiorQC LLC, January 28, 2016

11. *A Comparison of Collision Avoidance Calculations*
    Shola Okewummi, Andrew Brooks SPE/IADC-140183

12. *Borehole Position Uncertainty; Analysis of Measuring Methods and Derivation of Systematic Error Model*
    Chris J.M. Wolff and John P. DeWardt SPE-9223

13. *Directional Surveying: Rotating and Sliding Operations Give Different Wellbore Position Accuracy*
    Torgeir Torkildsen, Jon Bang, SPE 63275-MS, SPE ATCE, 1-4 October 2000, Dallas, Texas
11 List of Error Sources and Weighting Functions

Refer to section 4.2 for a details of the notation used below.

11.1 Error Sources Common to Both Gyro and MWD Models

Terms common to both gyro and MWD models:

<table>
<thead>
<tr>
<th>Error Code</th>
<th>Description</th>
<th>Propagation Mode</th>
<th>MD</th>
<th>Inc</th>
<th>Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>XYM1</td>
<td>xy misalignment 1</td>
<td>S/R</td>
<td>0</td>
<td>w₁₂</td>
</tr>
<tr>
<td>2</td>
<td>XYM2</td>
<td>xy misalignment 2</td>
<td>S/R</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>XYM3</td>
<td>xy misalignment 3</td>
<td>S/R</td>
<td>0</td>
<td>w₃₄cosA</td>
</tr>
<tr>
<td>4</td>
<td>XYM4</td>
<td>xy misalignment 4</td>
<td>S/R</td>
<td>0</td>
<td>w₃₄sinA</td>
</tr>
<tr>
<td>5</td>
<td>SAG / VSAG</td>
<td>Vertical sag (SAG in MWD model)</td>
<td>S</td>
<td>0</td>
<td>sinI</td>
</tr>
<tr>
<td>6</td>
<td>DRF-R</td>
<td>Depth random error</td>
<td>R</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>DRF-S</td>
<td>Depth systematic reference</td>
<td>S</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>DSF-W</td>
<td>Depth scale</td>
<td>S/W</td>
<td>ΔD</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>DST-G</td>
<td>Depth stretch type</td>
<td>G</td>
<td>(Dᵥ+DcosI). ΔD</td>
<td>0</td>
</tr>
</tbody>
</table>

11.2 MWD Error Sources

<table>
<thead>
<tr>
<th>Error Code</th>
<th>Description</th>
<th>Propagation Mode</th>
<th>MD</th>
<th>Inc</th>
<th>Weighting Function</th>
<th>Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>ABXY-TI1</td>
<td>Accelerometer bias – term1</td>
<td>S/R</td>
<td>0</td>
<td>- cosl/G</td>
<td>tanθ cosl sinAₘ/G</td>
</tr>
<tr>
<td>11</td>
<td>ABXY-TI2</td>
<td>Accelerometer bias – term2 (singular when vertical)</td>
<td>S/R</td>
<td>0</td>
<td>0</td>
<td>cot l – tanθ cosAₘ/G</td>
</tr>
<tr>
<td>12</td>
<td>ABZ</td>
<td>Accelerometer bias z-axis</td>
<td>S</td>
<td>0</td>
<td>- sinl/G</td>
<td>tanθ sinl sinAₘ/G</td>
</tr>
</tbody>
</table>

Definition of ISCWSA Error Model Rev4 .3
<table>
<thead>
<tr>
<th>No.</th>
<th>Section</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>ASXY-TI1</td>
<td>Accelerometer scale factor – term 1</td>
<td>$S \ 0 \ \frac{\sin \theta \cos \theta}{\sqrt{2}} \ - \frac{\tan \theta \sin \theta \cos \theta \sin \alpha_m}{\sqrt{2}}$</td>
</tr>
<tr>
<td>14</td>
<td>ASXY-TI2</td>
<td>Accelerometer scale factor – term 2</td>
<td>$S/R \ 0 \ \frac{\sin \theta \cos \theta}{\sqrt{2}} \ - \frac{\tan \theta \sin \theta \cos \theta \sin \alpha_m}{2}$</td>
</tr>
<tr>
<td>15</td>
<td>ASXY-TI3</td>
<td>Accelerometer scale factor – term 3</td>
<td>$S/R \ 0 \ 0 \ \frac{\tan \theta \sin \theta \cos \theta \sin \alpha_m}{2}$</td>
</tr>
<tr>
<td>16</td>
<td>ASZ</td>
<td>Accelerometer scale factor z-axis</td>
<td>$S \ 0 \ -\sin \theta \cos \theta$</td>
</tr>
<tr>
<td>17</td>
<td>MBXY-TI1</td>
<td>Magnetometer bias – term 1</td>
<td>$S/R \ 0 \ 0 \ -\frac{\cos \theta \sin \alpha_m}{\cos \alpha_m \ \frac{\cos \theta}{\cos \alpha_m}}$</td>
</tr>
<tr>
<td>18</td>
<td>MBXY-TI2</td>
<td>Magnetometer bias – term 2</td>
<td>$S/R \ 0 \ 0 \ -\frac{\cos \alpha_m}{\frac{\cos \theta}{\cos \alpha_m}}$</td>
</tr>
<tr>
<td>19</td>
<td>MBZ</td>
<td>Magnetometer bias z-axis</td>
<td>$S \ 0 \ 0 \ -\sin \theta \sin \alpha_m \ \frac{\cos \theta}{\cos \alpha_m}$</td>
</tr>
<tr>
<td>20</td>
<td>MSXY-TI1</td>
<td>Magnetometer scale factor – term 1</td>
<td>$S \ 0 \ 0 \ \frac{\sin \alpha_m (\tan \theta \cos \theta + \sin \theta \cos \alpha_m)}{\sqrt{2}}$</td>
</tr>
<tr>
<td>21</td>
<td>MSXY-TI2</td>
<td>Magnetometer scale factor – term 2</td>
<td>$S/R \ 0 \ 0 \ \frac{\sin \alpha_m (\tan \theta \cos \theta - \cos^2 \theta \cos \alpha_m - \cos \alpha_m)}{\sqrt{2}}$</td>
</tr>
<tr>
<td>22</td>
<td>MSXY-TI3</td>
<td>Magnetometer scale factor – term 3</td>
<td>$S/R \ 0 \ 0 \ \frac{(\cos \theta \cos^2 \alpha_m - \cos \theta \sin^2 \alpha_m - \tan \theta \sin \theta \cos \alpha_m)}{\sqrt{2}}$</td>
</tr>
<tr>
<td>23</td>
<td>MSZ</td>
<td>Magnetometer scale factor z-axis</td>
<td>$S \ 0 \ 0 \ -(\sin \theta \cos \alpha_m + \tan \theta \cos \theta) \sin \theta \sin \alpha_m$</td>
</tr>
<tr>
<td>24</td>
<td>AMIL</td>
<td>Axial magnetic interference</td>
<td>$S \ 0 \ 0 \ -\frac{\sin \alpha_m}{\cos \theta}$</td>
</tr>
<tr>
<td>25</td>
<td>ABIXY-TI1</td>
<td>Accelerometer bias – axial interference correction – term 1</td>
<td>$S/R \ 0 \ -\frac{\cos \theta}{G}$</td>
</tr>
<tr>
<td>26</td>
<td>ABIXY-TI2</td>
<td>Accelerometer bias – axial interference correction – term 2 (singular when vertical)</td>
<td>$S/R \ 0 \ -\frac{(\tan \theta \cos \alpha_m - \cot \theta \cos \theta)}{G(1 - \sin^2 \theta \sin^2 \alpha_m)}$</td>
</tr>
<tr>
<td>27</td>
<td>ABIZ</td>
<td>Accelerometer bias z-axis when axial interference correction applied</td>
<td>$S \ 0 \ -\frac{\sin \theta \cos \alpha_m}{G} \ \frac{(\tan \theta \cos \theta + \sin \theta \cos \alpha_m)}{G(1 - \sin^2 \theta \sin^2 \alpha_m)}$</td>
</tr>
<tr>
<td>28</td>
<td>ASIXY-TI1</td>
<td>Accelerometer scale factor – axial interference correction – term 1</td>
<td>$S \ 0 \ \frac{\sin \theta \cos \alpha_m}{\sqrt{2}} \ \frac{(\tan \theta \cos \theta + \sin \theta \cos \alpha_m)}{\sqrt{2}(1 - \sin^2 \theta \sin^2 \alpha_m)}$</td>
</tr>
<tr>
<td>29</td>
<td>ASIXY-TI2</td>
<td>Accelerometer scale factor – axial interference correction – term 2</td>
<td>$S/R \ 0 \ \frac{\sin \theta \cos \alpha_m}{2} \ \frac{(\tan \theta \cos \theta + \sin \theta \cos \alpha_m)}{2(1 - \sin^2 \theta \sin^2 \alpha_m)}$</td>
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</tr>
<tr>
<td>30</td>
<td>ASIXY-TI3</td>
<td>Accelerometer scale factor – axial interference correction – term3</td>
<td>S/R</td>
</tr>
<tr>
<td>31</td>
<td>ASIZ</td>
<td>Accelerometer scalefactor z-axis when axial interference correction applied</td>
<td>S</td>
</tr>
<tr>
<td>32</td>
<td>MBIXY-TI1</td>
<td>Magnetometer bias – axial interference correction – term1</td>
<td>S/R</td>
</tr>
<tr>
<td>33</td>
<td>MBIXY-TI2</td>
<td>Magnetometer bias – axial interference correction – term2</td>
<td>S/R</td>
</tr>
<tr>
<td>34</td>
<td>MSIXY-TI1</td>
<td>Magnetometer scale factor – axial interference correction – term1</td>
<td>S</td>
</tr>
<tr>
<td>35</td>
<td>MSIXY-TI2</td>
<td>Magnetometer scale factor – axial interference correction – term2</td>
<td>S/R</td>
</tr>
<tr>
<td>36</td>
<td>MSIXY-TI3</td>
<td>Magnetometer scale factor – axial interference correction – term3</td>
<td>S/R</td>
</tr>
<tr>
<td>37</td>
<td>DEC</td>
<td>Constant declination error</td>
<td>G/R</td>
</tr>
<tr>
<td>38</td>
<td>DBH</td>
<td>Declination error dependant on the horizontal component of Earth’s field</td>
<td>G/R</td>
</tr>
<tr>
<td>39</td>
<td>MFI</td>
<td>Earth’s total magnetic field when axial interference correction applied</td>
<td>G/R</td>
</tr>
<tr>
<td>40</td>
<td>MDI</td>
<td>Dip angle when axial interference correction applied</td>
<td>G/R</td>
</tr>
</tbody>
</table>
11.3 Gyro Error Sources

For the gyro model, the error sources may be grouped depending on the tool sensor configuration and further split into those which apply in Stationary survey mode, Continuous survey mode or either mode. During a single survey leg a tool made transition between these modes as a function of inclination.

<table>
<thead>
<tr>
<th>Error Code</th>
<th>Description</th>
<th>Survey Mode</th>
<th>Propagation Mode</th>
<th>Weighting Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>AXYZ-XYB 3-axis: xy accelerometer bias</td>
<td>C/S</td>
<td>S/R</td>
<td>MD 0 cos I / G 0 Azimuth</td>
</tr>
<tr>
<td>42</td>
<td>AXYZ-ZB 3-axis: z accelerometer bias</td>
<td>C/S</td>
<td>S</td>
<td>Inc sin I / G 0 Azimuth</td>
</tr>
<tr>
<td>43</td>
<td>AXYZ-SF 3-axis: accelerometer scale factor error</td>
<td>C/S</td>
<td>S</td>
<td>1 3 sin I cos I 0 Azimuth</td>
</tr>
<tr>
<td>44</td>
<td>AXYZ-MIS 3-axis: accelerometer misalignment</td>
<td>C/S</td>
<td>S</td>
<td>1 0 Azimuth</td>
</tr>
<tr>
<td>45</td>
<td>AXY-B 2-axis: xy accelerometer bias</td>
<td>C/S</td>
<td>S/R</td>
<td>G cos(I - k y) 0 Azimuth</td>
</tr>
<tr>
<td>46</td>
<td>AXY-SF 2-axis: Accelerometer scale factor error</td>
<td>C/S</td>
<td>S</td>
<td>tan(I - k y) 0 Azimuth</td>
</tr>
<tr>
<td>47</td>
<td>AXY-MS 2-axis: Accelerometer misalignment</td>
<td>C/S</td>
<td>S</td>
<td>1 0 Azimuth</td>
</tr>
<tr>
<td>48</td>
<td>AXY-GB 2-axis: Gravity Bias</td>
<td>C/S</td>
<td>S</td>
<td>tan(I - k y) / G 0 Azimuth</td>
</tr>
<tr>
<td>49</td>
<td>GXYZ-XYB1 3-axis, stationary: xy gyro bias 1</td>
<td>S</td>
<td>S/R</td>
<td>sinA_f cos I / cos I sin I cos I</td>
</tr>
<tr>
<td>50</td>
<td>GXYZ-XYB2 3-axis, stationary: xy gyro bias 2</td>
<td>S</td>
<td>S/R</td>
<td>cos A_f / cos I sin I cos I</td>
</tr>
<tr>
<td>51</td>
<td>GXYZ-XYRN 3-axis, stationary: xy gyro random noise</td>
<td>S</td>
<td>R</td>
<td>f \sqrt{1 - sin^2 A_f sin^2 I} / cos I sin I cos I</td>
</tr>
<tr>
<td>52</td>
<td>GXYZ-XYG1 3-axis, stationary: xy gyro g-dependent error 1</td>
<td>S</td>
<td>S</td>
<td>cos A_f sin I / cos I sin I cos I</td>
</tr>
<tr>
<td>53</td>
<td>GXYZ-XYG2 3-axis, stationary: xy gyro g-dependent error 2</td>
<td>S</td>
<td>S/R</td>
<td>cos A_f cos I / cos I sin I cos I</td>
</tr>
<tr>
<td>54</td>
<td>GXYZ-XYG3 3-axis, stationary: xy gyro g-dependent error 3</td>
<td>S</td>
<td>S/R</td>
<td>sin A_f cos^2 I / cos I sin I cos I</td>
</tr>
<tr>
<td>55</td>
<td>GXYZ-XYG4 3-axis, stationary: xy gyro g-dependent error 4</td>
<td>S</td>
<td>S</td>
<td>sin A_f sin I cos I / cos I sin I</td>
</tr>
<tr>
<td>56</td>
<td>GXYZ-ZB 3-axis, stationary: z gyro bias</td>
<td>S</td>
<td>S</td>
<td>sin A_f sin I / cos I sin I cos I</td>
</tr>
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</tr>
<tr>
<td>57</td>
<td>GXYZ-ZRN</td>
<td>3-axis, stationary: z gyro random noise</td>
<td>S</td>
<td>R</td>
</tr>
<tr>
<td>58</td>
<td>GXYZ-ZG1</td>
<td>3-axis, stationary: z gyro g-dependent error 1</td>
<td>S</td>
<td>S/R</td>
</tr>
<tr>
<td>59</td>
<td>GXYZ-ZG2</td>
<td>3-axis, stationary: z gyro g-dependent error 2</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>60</td>
<td>GXYZ-SF</td>
<td>3-axis, stationary: Gyro scalefactor</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>61</td>
<td>GXYZ-MIS</td>
<td>3-axis, stationary: Gyro misalignment</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>62</td>
<td>GXY-B1</td>
<td>2-axis, stationary: xy gyro bias 1</td>
<td>S</td>
<td>S/R</td>
</tr>
<tr>
<td>63</td>
<td>GXY-B2</td>
<td>2-axis, stationary: xy gyro bias 2</td>
<td>S</td>
<td>S/R</td>
</tr>
<tr>
<td>64</td>
<td>GXY-RN</td>
<td>2-axis, stationary: xy gyro random noise</td>
<td>S</td>
<td>R</td>
</tr>
<tr>
<td>65</td>
<td>GXY-G1</td>
<td>2-axis, stationary: xy gyro g-dependent error 1</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>66</td>
<td>GXY-G2</td>
<td>2-axis, stationary: xy gyro g-dependent error 2</td>
<td>S</td>
<td>S/R</td>
</tr>
<tr>
<td>67</td>
<td>GXY-G3</td>
<td>2-axis, stationary: xy gyro g-dependent error 3</td>
<td>S</td>
<td>S/R</td>
</tr>
<tr>
<td>68</td>
<td>GXY-G4</td>
<td>2-axis, stationary: xy gyro g-dependent error 4</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>69</td>
<td>GXY-SF</td>
<td>2-axis, stationary: Gyro scalefactor</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>70</td>
<td>GXY-MIS</td>
<td>2-axis, stationary: Gyro misalignment</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>71</td>
<td>EXT-REF</td>
<td>External reference error</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>72</td>
<td>EXT-TIE</td>
<td>Un-modelled random azimuth error in tie-on tool</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>73</td>
<td>EXT-MIS</td>
<td>Misalignment effect at tie-on</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>74</td>
<td>GXYZ-GD</td>
<td>3-axis, continuous: xyz gyro drift</td>
<td>C</td>
<td>S</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
<td>75</td>
<td>GXYZ-RW</td>
<td>3-axis, continuous: xyz gyro random walk</td>
<td>C</td>
<td>S</td>
</tr>
<tr>
<td>76</td>
<td>GXY-GD</td>
<td>2-axis, continuous: xy gyro drift</td>
<td>C</td>
<td>S</td>
</tr>
<tr>
<td>77</td>
<td>GXY-RW</td>
<td>2-axis, continuous: xy gyro random walk</td>
<td>C</td>
<td>S</td>
</tr>
<tr>
<td>78</td>
<td>GZ-GD</td>
<td>z-axis, continuous: z gyro drift</td>
<td>C</td>
<td>S</td>
</tr>
<tr>
<td>79</td>
<td>GZ-RW</td>
<td>z-axis, continuous: z gyro random walk</td>
<td>C</td>
<td>S</td>
</tr>
</tbody>
</table>

Note when the sensor are rotated then weighting functions may reduce to zero. This applies as follows:

- $= 0$ when xy sensors are z rotated
- Inclination function source 41;
- Azimuth function sources 49, 50, 58, 62, 63.

- $= 0$ when xy sensors are z rotated and gamma=0°
- Inclination function, source 45.

- $= 0$ when z sensor is x(y) rotated
- Inclination function source 42.

Refer to [2] for further details.
11.4 Utility Sources
The following sources do not represent any clear physical source of error, in Blind and Unknown tool modelling to guestimate position uncertainty. Alternatively, the effect of CNA and CNI can be represented using misalignment terms.

| 80 | CNA | Linear Cone – Inclination | C | S | 0 | 0 | \( h_i = \frac{1}{\sin I} \) |
| 81 | CNI | Linear Cone - Azimuth     | C | S | 0 | 1 | 0 |

11.5 Vertical Singularities
Several of the functions above are singular in vertical hole. The following formula may be substituted when vertical.

| 3v | XYM3 | xy misalignment 3 \((\text{singular when vertical})\) | North Formula | East Formula | Vertical Formula |
| 4v | XYM4 | xy misalignment 4 \((\text{singular when vertical})\) | 1 | 0 | 0 |
| 11v | ABXY-TI2 | Accelerometer bias – term 2 \((\text{singular when vertical})\) | \(-\sin A_m/G\) | \(\cos A_m/G\) | 0 |
| 20v | ABIXY-TI2 | Accelerometer bias – axial interference correction – term 2 \((\text{singular when vertical})\) | \(-\sin A_m/G\) | \(\cos A_m/G\) | 0 |
| 80v | CNA | Linear Cone – Inclination \((\text{singular when vertical})\) | \(-\sin(Az)\) | \(\cos(Az)\) | 0 |

Note. XYM2 is also singular when vertical is misalignment option 1 is used. However as noted in the [2] in this situation this term may give strange/unwanted values when azimuth or toolface vary.
11.6 Historic Terms: No Longer Used in the MWD Model After Revisions 3

See section §3.4 for a discussion of the revisions to the MWD model. The following weighting functions have been replaced by new methods introduced in revision 1 (misalignment terms MX and MY replaced), revision 3 (toolface dependant terms) and revision 4 (AMIL replaces AMIC and AMID for drill string interference).

<table>
<thead>
<tr>
<th>Error Code</th>
<th>Description</th>
<th>Weighting Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MX</td>
<td>Tool axial misalignment – x-axis</td>
<td>MD = 0, Inc = sin α, Azimuth = -cos α / sin l</td>
</tr>
<tr>
<td>2 MY</td>
<td>Tool axial misalignment – y-axis</td>
<td>MD = 0, Inc = cos α</td>
</tr>
<tr>
<td>3 ABX</td>
<td>Accelerometer bias x-axis</td>
<td>MD = 0, Inc = -cos l / sin a, Azimuth = (cos l sin A_m sin a - cos A_m cos a) tan θ + cot l cos a / G</td>
</tr>
<tr>
<td>4 ABY</td>
<td>Accelerometer bias y-axis</td>
<td>MD = 0, Inc = -cos l / sin a, Azimuth = (cos l sin A_m cos a + cos A_m sin a) tan θ - cot l sin a / G</td>
</tr>
<tr>
<td>5 ASX</td>
<td>Accelerometer scalefactor x-axis</td>
<td>MD = 0, Inc = sin l cos l sin^2 a, Azimuth = -(tan θ sin l (cos l sin A_m sin a - cos A_m cos a) + cos l cos a) sin a / G</td>
</tr>
<tr>
<td>6 ASY</td>
<td>Accelerometer scalefactor y-axis</td>
<td>MD = 0, Inc = sin l cos l cos^2 a, Azimuth = -(tan θ sin l (cos l sin A_m cos a + cos A_m sin a) - cos l sin a) cos a / G</td>
</tr>
<tr>
<td>7 MBX</td>
<td>Magnetometer bias x-axis</td>
<td>MD = 0, Inc = 0, Azimuth = cos A_m cos a - cos l sin A_m sin a / G</td>
</tr>
<tr>
<td>8 MBY</td>
<td>Magnetometer bias y-axis</td>
<td>MD = 0, Inc = 0, Azimuth = -cos A_m sin a + cos l sin A_m cos a / G</td>
</tr>
<tr>
<td>9 MSX</td>
<td>Magnetometer scalefactor x-axis</td>
<td>MD = 0, Inc = 0, Azimuth = (cos l cos A_m sin a - tan θ sin l sin A_m + sin A_m cos a) (cos A_m cos a - cos l sin A_m sin a) / G</td>
</tr>
<tr>
<td>10 MSY</td>
<td>Magnetometer scalefactor y-axis</td>
<td>MD = 0, Inc = 0, Azimuth = -(cos l cos A_m cos a - tan θ sin l sin a - sin A_m sin a) (cos A_m sin a + cos l sin A_m cos a) / G</td>
</tr>
<tr>
<td>11 ABIX</td>
<td>Accelerometer bias x-axis when axial interference correction applied.</td>
<td>MD = 0, Inc = -cos l / sin a, Azimuth = cos^2 l sin A_m sin a (tan θ cos l + sin l cos A_m) - cos a (tan θ cos A_m - cot l) / G (1 - sin^2 l sin^2 A_m)</td>
</tr>
<tr>
<td>12 ABY</td>
<td>Accelerometer bias y-axis when axial interference correction applied.</td>
<td>MD = 0, Inc = -cos l / sin a, Azimuth = cos^2 l sin A_m cos a (tan θ cos l + sin l cos A_m) + sin a (tan θ cos A_m - cot l) / G (1 - sin^2 l sin^2 A_m)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Accelerometer scalefactor x-axis when axial interference correction applied.</td>
</tr>
<tr>
<td>---</td>
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</tr>
<tr>
<td></td>
<td>ASIX</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ASIY</td>
<td>Accelerometer scalefactor y-axis when axial interference correction applied.</td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>MBIX</td>
<td>Magnetometer bias x-axis when axial interference correction applied.</td>
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</tr>
<tr>
<td></td>
<td>MBIY</td>
<td>Magnetometer bias y-axis when axial interference correction applied.</td>
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<td>MSIX</td>
<td>Magnetometer scalefactor x-axis when axial interference correction applied.</td>
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<td>Constant axial magnetic interference</td>
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<td>AMID</td>
<td>Direction dependant axial magnetic interference</td>
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