Survey Uncertainty Quantification with R: Need for an Explicit Definition of the Chi-Square Tests

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Overview

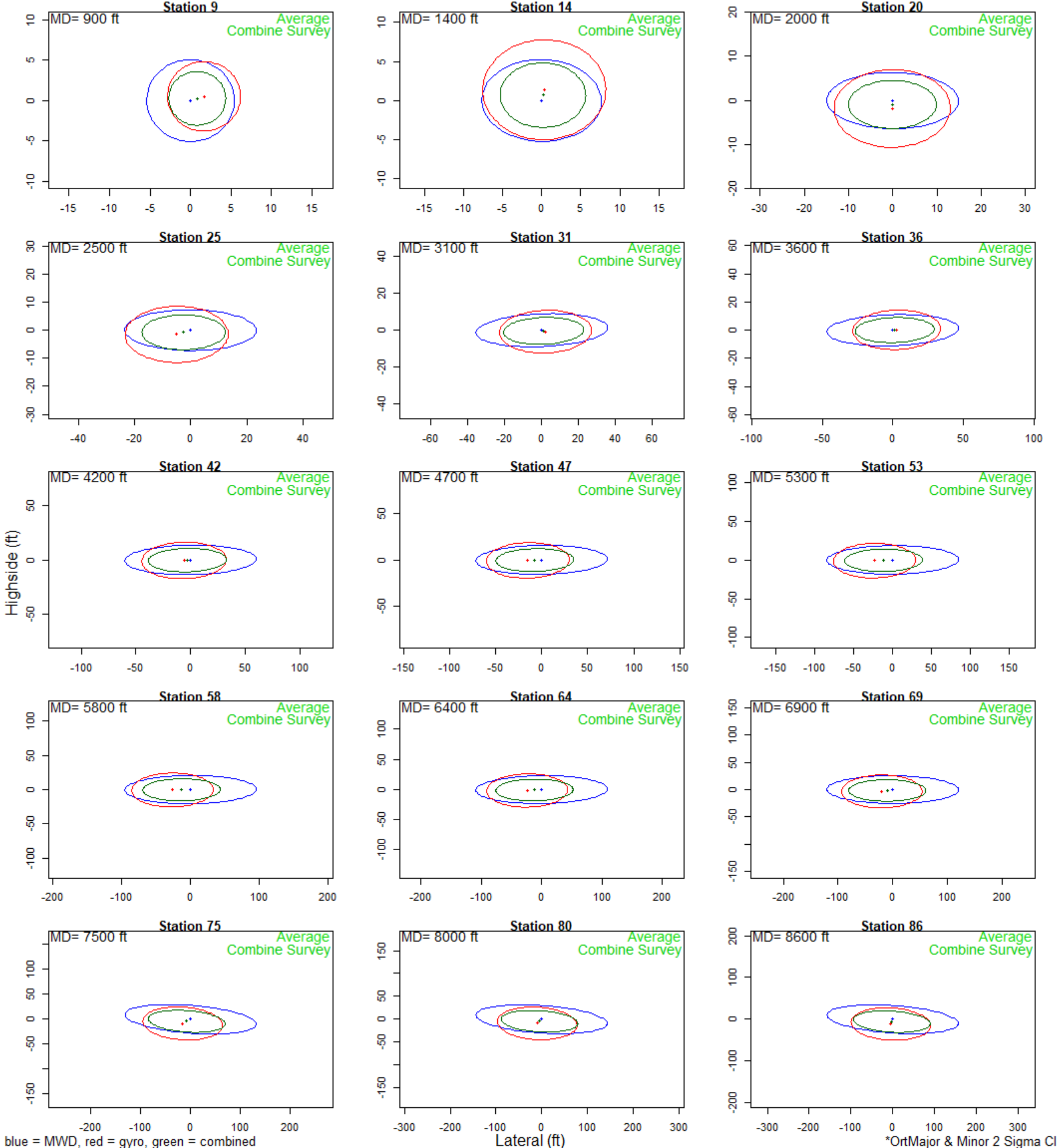
1. ***Why?***
2. Combined Survey Project
3. Common Survey QC Tests
   a. Qualitative Ellipse Visual Tests
   b. RIP Test
   c. Chi-Squared Tests
      1. One Sided for Individual Wells
      2. Two Sided for EM Validation & Refinement
4. Current Chi-Square Test Implementation per Ekseth *et al.*, 2007 (SPE-105558)
   a) Limitations, Assumptions, & Concerns
   b) Need to explicitly define all QC Tests so they can be run correctly and consistently
5. Overview of R and preview of current QC Report code(slides to be posted)

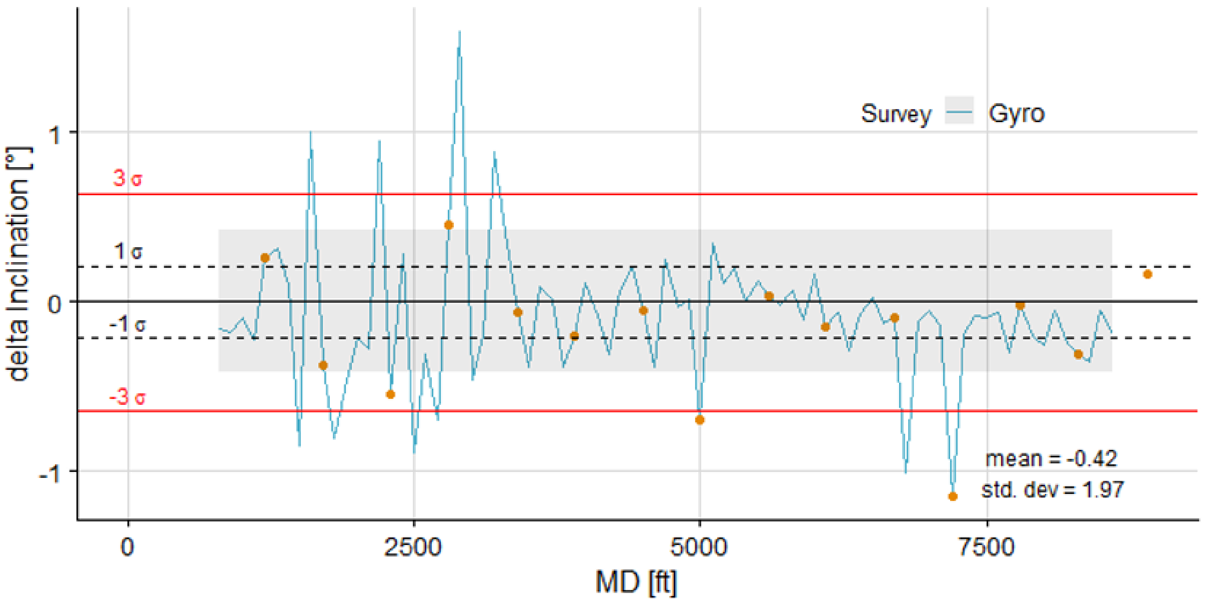
Why?

1. To **explicitly define uncertainty expectations** for survey data and the **means to determine** when a tool is not performing as assumed by the EMs
   - **ISCWSA OWSG Mission Statement**: To promote practices that provide confidence that reported positions are within their stated uncertainty

2. "To **obtain the maximum amount of useful information from the data on hand** without being able to repeat the experiment with better equipment or reduce statistical uncertainty by making more measurements"

   - Bevington, Data Reduction and Error Analysis for the Physical Sciences





Shaded area = Tolerance, orange dots = 15 stations used for the Chi-Square Test

Table 2: Result of all Chi Square (X^2) tests

	X^2 Test Value	Test Limit	Test Conclusion
IDT	60.19	34.4	Fail
ADT	27.88	34.4	Pass
CODT (HLA)	-	-	Pass
X_L	1.01	34.4	Pass
X_H	0.97	34.4	Pass
X_W	0.29	34.4	Pass

QC Test Overview – SPE-212492

- Ellipse Test
- RIP Test
- Chi-Square Tests (IDT, ADT, CODT)

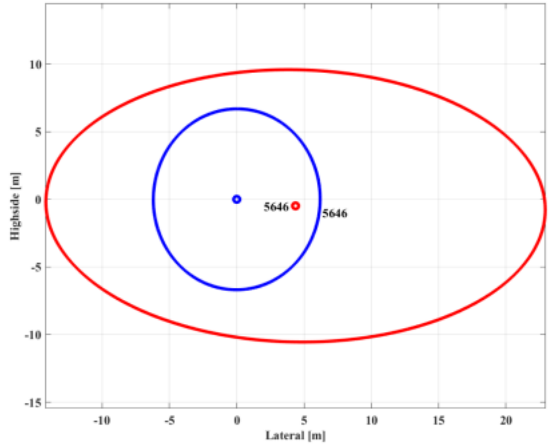


Figure 3—Comparison of GWD OMM (blue) and MWD (red) uncertainty ellipses at 5646m.

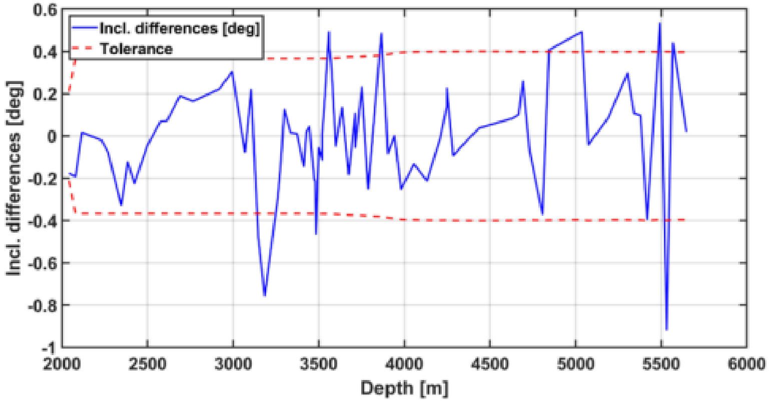


Figure 1—Inclination RIP test between GWD OMM × MWD

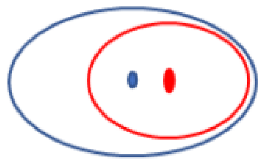
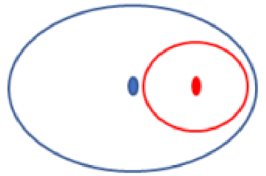
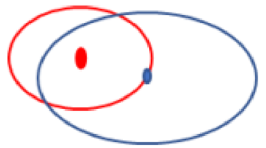
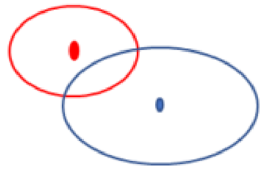
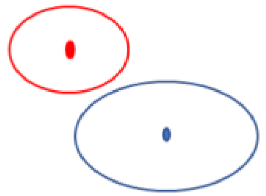
$$\chi = \sum_{i=1}^{n} \frac{\Delta x_i^{\,2}}{(\sigma_{1,i}^{\;2} + \sigma_{2,i}^{\;2})} \leq Z_{\gamma,n}$$

x: inclination, azimuth, or CODT (highside/lateral/or along-hole)

Ellipse Test

- Visual test based on overlap of surveys center point and uncertainty levels
  - Scaled to what CI???
    - **2 & 3 sigma feedback**
  - NEV or HLA ref frames?
- Results found here seem comparable to CODT Chi-Square Test if n<5
- A 1 sigma CI would be the most conservative definition for this test

Table 5—Ellipsis of Uncertainty for Survey Quality Analysis

Level Agreement	Description of Agreement level	Action	Pictorial Description of Agreement Level
Very Good	MWD ellipse fully encompasses gyro ellipse, and gyro ellipse encompasses centre of MWD ellipse.	No further investigation needed.	
Good	MWD ellipse fully encompasses gyro ellipse, but gyro ellipse does not encompass centre of MWD ellipse.	No further investigation needed.	
Average	MWD ellipse dows not fully encompass gyro ellipse but overlaps with it. The center of the gyro ellipse lies inside the MWD ellipse.	No further investigation needed.	
Poor	MWD ellipse dows not fully encompass gyro ellipse but overlaps with it. The centre of the gyro ellipse lies outside the MWD ellipse.	Investigate – if unresolved consider re-survey.	
Unacceptable	Ellipses do not overlap.	Probably re-survey immediately and investigate.	

Ellipse Test Improvement?

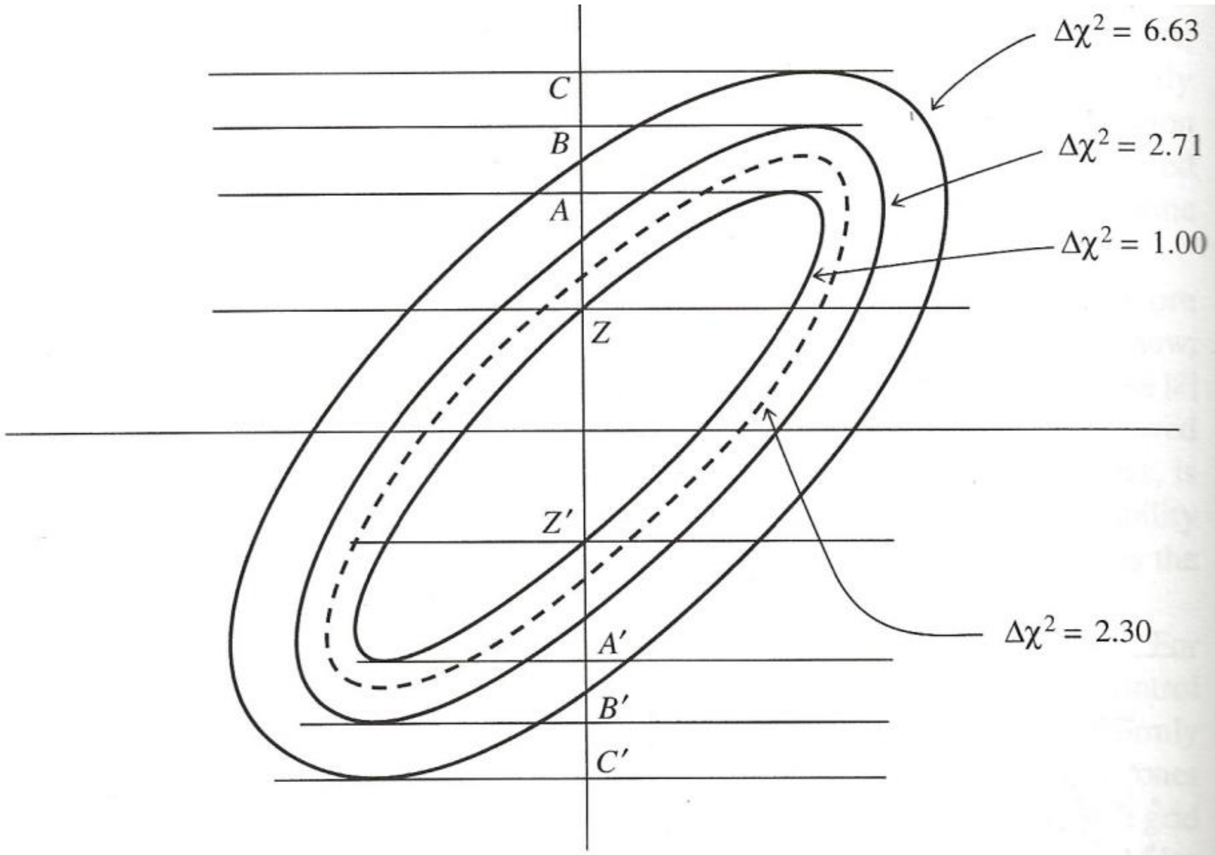


Figure 15.6.4. Confidence region ellipses corresponding to values of chi-square larger than the fitted minimum. The solid curves, with $\Delta\chi^2 = 1.00, 2.71, 6.63$ project onto one-dimensional intervals AA', BB', CC'. These intervals — not the ellipses themselves — contain 68.3%, 90%, and 99% of normally distributed data. The ellipse that contains 68.3% of normally distributed data is shown dashed, and has $\Delta\chi^2 = 2.30$. For additional numerical values, see accompanying table.

Press, W. H., B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes, The Art of Scientific Computing,* Cambridge University Press, New York (1986).

Table 5—Ellipsis of Uncertainty for Survey Quality Analysis

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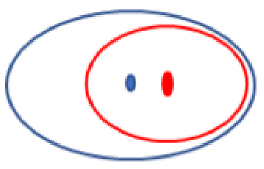
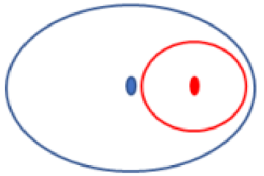
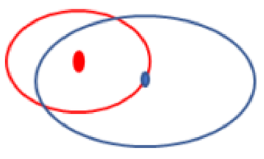
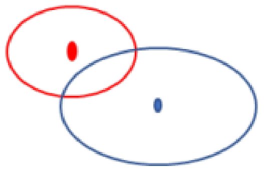
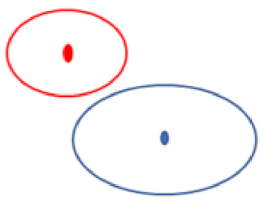
Very Good	MWD ellipse fully encompasses gyro ellipse, and gyro ellipse encompasses centre of MWD ellipse.	No further investigation needed.	
Good	MWD ellipse fully encompasses gyro ellipse, but gyro ellipse does not encompass centre of MWD ellipse.	No further investigation needed.	
Average	MWD ellipse dows not fully encompass gyro ellipse but overlaps with it. The center of the gyro ellipse lies inside the MWD ellipse.	No further investigation needed.	
Poor	MWD ellipse dows not fully encompass gyro ellipse but overlaps with it. The centre of the gyro ellipse lies outside the MWD ellipse.	Investigate – if unresolved consider re-survey.	
Unacceptable	Ellipses do not overlap.	Probably re-survey immediately and investigate.	

SPE-212492

Relative Instrument Performance (RIP) Test

- Compares mean and standard deviation differences (only inclination and azimuth; independently) by normalized differences ***(ISCWSA 56 Pres by Jerry Codling)***
  - RP-78 to clearly define this too?
  - Std.Dev assumption of n or n-1 should be stated
- Mean = systematic errors
- Standard deviation = random errors
- Results are levels of agreement based on the mean/standard deviation numeric differences

Table 4—RIP test tolerances – Adapted from IADC/SPE 199554

Mean Tolerances	STD Tolerances	Results
abs(mean_diff) <= 0.50	std_diff <= 1.00	Good agreement
abs(mean_diff) <= 0.75	std_diff <= 1.50	Average agreement
abs(mean_diff) <= 1.25	std_diff <= 2.00	Poor agreement
abs(mean_diff) > 1.25	std_diff > 2.00	Disagreement

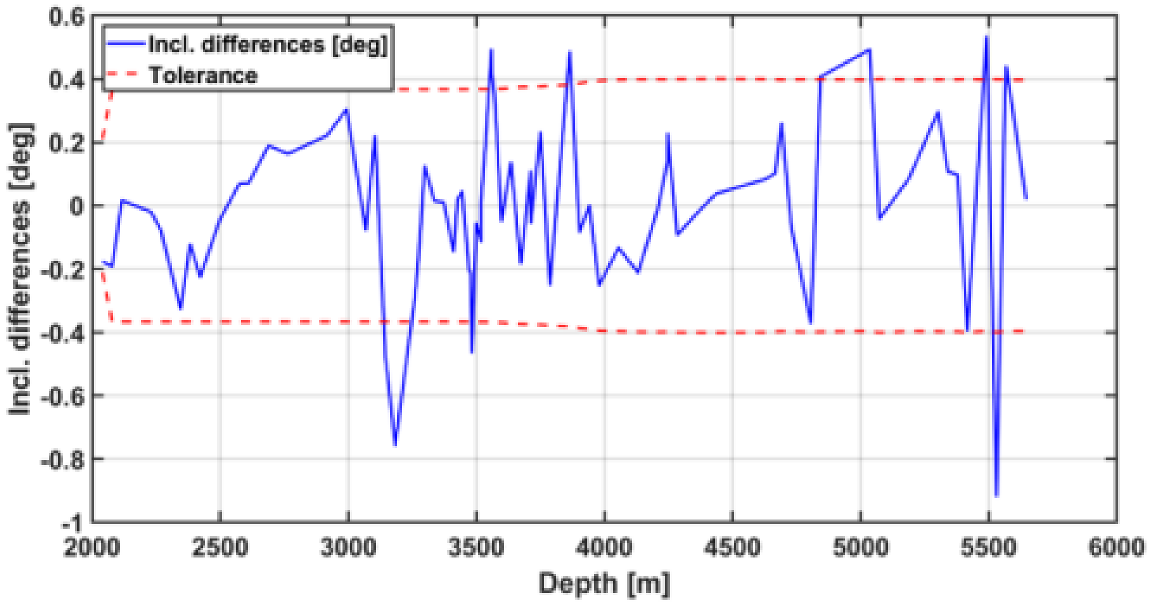
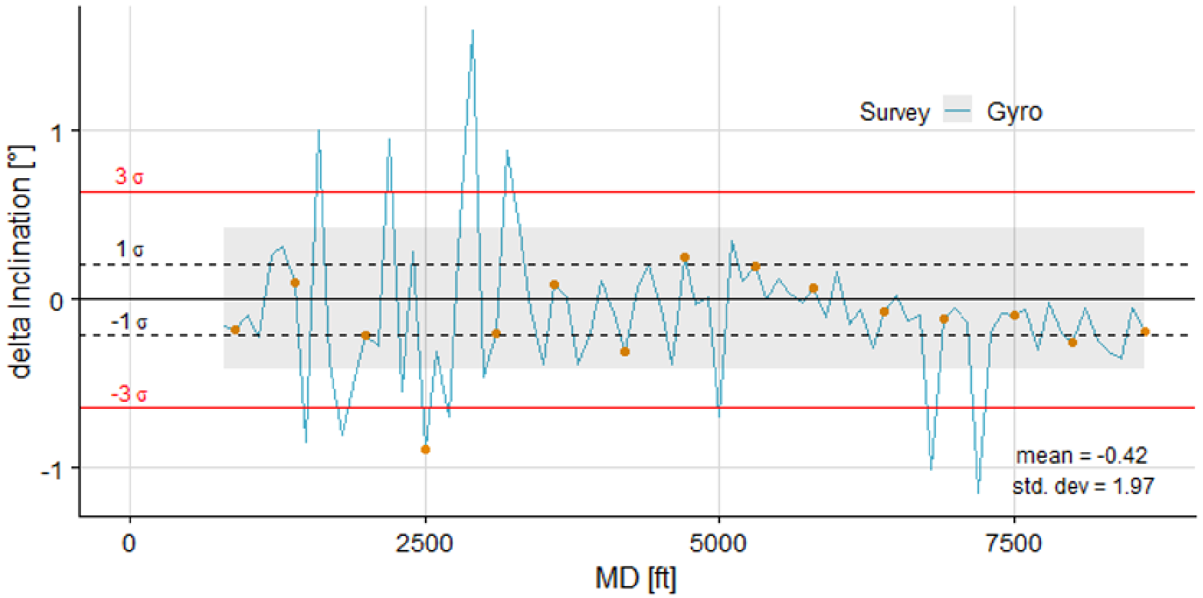


Figure 1—Inclination RIP test between GWD OMM × MWD



Shaded area = Tolerance, orange dots = 15 stations used for the Chi-Square Test

American Society for Quality(ASQ) – Control Chart

Out-of-control signals

- A single point outside the control limits. In Figure 1, point sixteen is above the UCL (upper control limit).
- 2 out of 3 successive points are on the same side of the centerline and farther than 2 σ from it. In Figure 1, point 4 sends that signal.
- 4 out of the 5 successive points are on the same side of the centerline and farther than 1 σ from it. In Figure 1, point 11 sends that signal.
- A run of 8 in a row are on the same side of the centerline. Or 10 out of 11, 12 out of 14, or 16 out of 20. In Figure 1, point 21 is 8th in a row above the centerline.
- Obvious consistent or persistent patterns that suggest something unusual about your data and your process.

*When you start a new control chart, the process may be out of control. If so, the control limits calculated from the first 20 points are conditional limits. When you have at least 20 sequential points from a period when the process is operating in control, recalculate control limits.

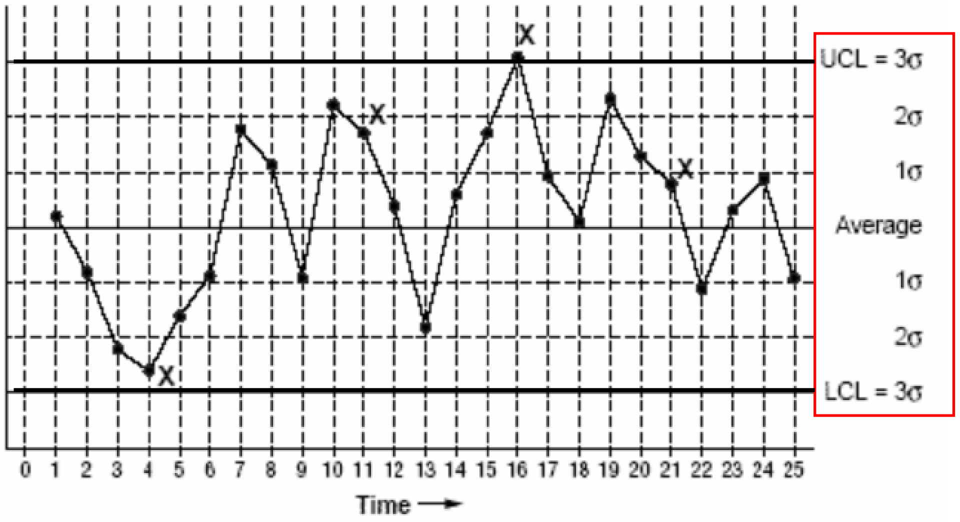
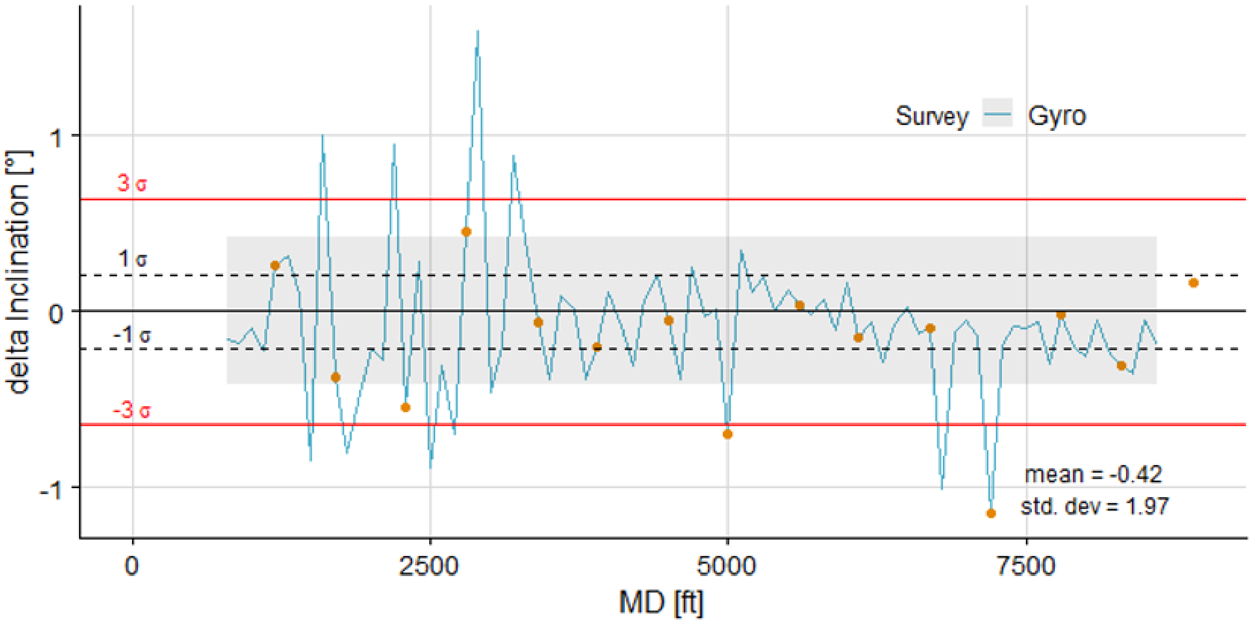


Figure 1 Control Chart: Out-of-Control Signals

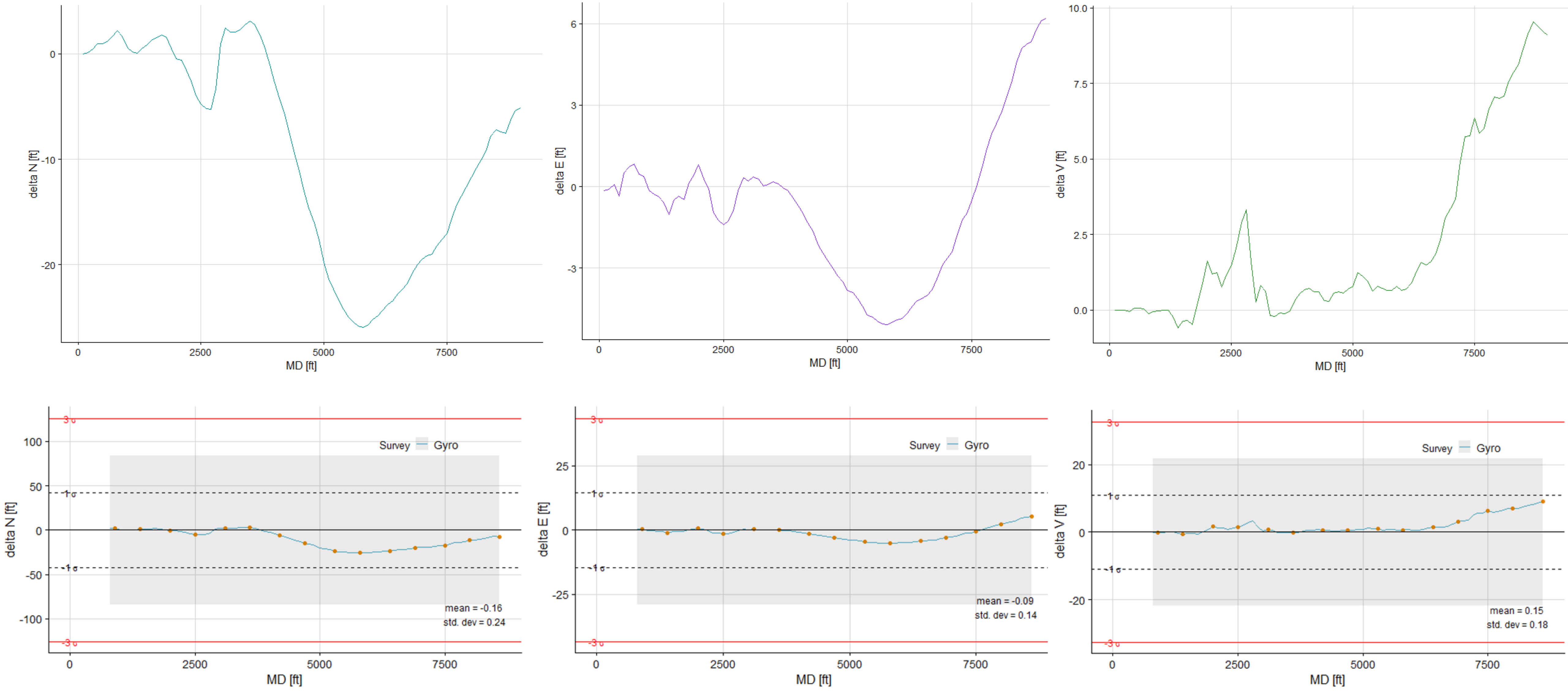
Current RIP/Control Chart Option from R



Shaded area = Tolerance, orange dots = 15 stations used for the Chi-Square Test

https://asq.org/quality-resources/control-chart

Distance RIP Plots – Improvement Idea



What is an Explicit Definition? – STDEV.P

Excel Definition and Function:

- Calculates standard deviation based on the entire population given as arguments (ignores logical values and text)
- The standard deviation is a measure of how widely values are dispersed from the average value (the mean).
- Assumptions: Arguments are the entire population (n).
  - If data is for a sample use (STDEV.S)
- For larger sample sizes, STDEV.P and STDEV.S can return ~ equal values
- Calculated using "n" method

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

Excel STDEV.P function

Formula	Description	Result
=STDEV.P(A3:A12)	Standard deviation of breaking strength, assuming only 10 tools are produced.	26.05455814

1345	1301	1368	1322	1310
1370	1318	1350	1303	1299

Using the above data results in a standard deviation (p) of 26.05

https://support.microsoft.com/en-us/office/stdev-p-function-6e917c05-31a0-496f-ade7-4f4e7462f285#:~:text=P%20function,-Excel%20for%20Microsoft&text=Calculates%20standard%20deviation%20based%20on,average%20value%20(the%20mean)

Chi-Square Test

- A normally distributed measurement and variance/uncertainty is transformed into a Chi-Square distributed measurement
- 5 Test Total
  - Inclination – IDT
  - Azimuth - ADT
  - 3 CODTs in HLA reference frame
    - NEV can be tested too, but HLA is preferred
- Results are compared with a test limit (Z)
  - Z value = number of stations (n) [15 stations is recommended] and significance level (γ)

$$X = \sum_{i=1}^{n} \frac{x_i^2}{\sigma_i^2} \le Z_{\gamma,n}$$

where $Z_{\gamma,n}$ is the Chi-square test limit for n degrees of freedom, at a significance level of γ. The significance level is, with one exception, fixed at 0.3% throughout this paper, in

SPE-105558 Eqn referenced above

1.0 Sigma Uncertainty/Scaled Variance Expectation Interpretation

Table 2: Result of all Chi Square (X^2) tests

	X^2 Test Value	Test Limit	Test Conclusion
IDT	27.98	34.4	Pass
ADT	18.39	34.4	Pass
CODT (HLA)	-	-	Pass
X_L	1.23	34.4	Pass
X_H	0.87	34.4	Pass
X_W	0.25	34.4	Pass

3.0 Sigma Uncertainty/Scaled Variance Expectation Interpretation

Table 2: Result of all Chi Square (X^2) tests

	X^2 Test Value	Test Limit	Test Conclusion
IDT	3.11	34.39	Pass
ADT	2.04	34.39	Pass
CODT (HLA)	-	-	Pass
X_L	0.14	34.39	Pass
X_H	0.1	34.39	Pass
X_W	0.03	34.39	Pass

Excel Test Limit Equation: CHISQ.INV.RT(0.003,15) = 34.4

Chi-Square Test (cont.)

- A Statistical Measure of ***Goodness-of-Fit***
  - Numerical Quantification of Agreement like a RIP Test
  - Hypothesis Testing – Does the survey disagreement exceed our EM expectation

- It tells us ***if differences*** in our sampling of observed surveys (INC, AZI, NEV/HLA Coordinates) ***are a reasonable expectation*** with the ***associated uncertainty at a given Confidence Interval***.

- It answers the following Questions:
  1. Do the discrepancies observed in overlapping surveys disprove one of the EM selections (i.e., is the EM selection optimistic) – One Sided Test
  2. Do our selections of observed surveys Fit within an expected range from the EM? - Two-Sided Test(SPE-199554)
     1. If $<$ lower bound, the EM is pessimistic to a given Probability
     2. If $>$ upper bound the EM is optimistic to a given Probability

- Do not make the mistake of concluding the IPM is “verified” or “proven”.

  “The curse of Statistics is that it can never prove things, only disprove them!”

  - Press, Numerical Recipes: The Art of Scientific Computing

Uncertainty Expectation – Test Decision

- How should our expected Variance or Uncertainty(std dev=sqrt(Variance)) sigma be calculated?
- **Not Explicitly Defined!**
  - **1 sigma seems too pessimistic(Prone to Type One Error – False Negative) for reasonable discrepancies**
  - 3 sigma may be too optimistic(Prone to Type Two Errors – False Positive)
- **Column 3 in Table 2** appears to show the average discrepancy/uncertainty ratio required to equal the Selected Test Limit
- Does an Ellipse Test scaled at 1.5 sigma make sense with Poor/Bad actions??

SPE/IADC 105558

High-Integrity Wellbore Surveys: Methods for Eliminating Gross Errors

Roger Ekseth, SPE, Gyrodata; Torgeir Torkildsen, SPE, Statoil ASA; Andrew Brooks, SPE, Baker Hughes Inteq; John Weston, SPE, Gyrodata; Erik Nyrnes, SPE, Statoil ASA; Harry Wilson, SPE, Baker Hughes Inteq; and Kazimir Kovalenko, SPE, Gyrodata

The Chi-square distribution statistical test. A Normally distributed measurement (x) with zero expectation and variance, σ^2, is transformed into an apparent one degree of freedom Chi-square distributed measurement by squaring the measurement and dividing by the variance. A given number (n) of one degree of freedom Chi-square distributed measurements, originating from n independent (uncorrelated) measurements, can be added together into a common Chi-square distributed test variable (X) with n degrees of freedom. The measurements can then be controlled against gross errors at a given confidence (γ) by testing if the following condition is fulfilled:

$$X = \sum_{i=1}^{n} \frac{x_i^2}{\sigma_i^2} \leq Z_{\gamma,n}$$

where $Z_{\gamma,n}$ is the Chi-square test limit for n degrees of freedom, at a significance level of γ. The significance level is, with one exception, fixed at 0.3% throughout this paper, in order to harmonise with the significance level used for the Normal distribution tests.

The Chi-square distribution test may be presented in an alternative form, when all n summed measurements have the same variance, σ^2.

Table 2: Chi-square distribution test limits and standard deviation scaling factors at a 0.3% significance level

n	$Z_{0.003,n}$	$\sqrt{Z_{0.003,n}/n}$
1	8.8	3.0
3	13.9	2.2
5	18.0	1.9
15	34.4	1.5
100	143	1.2
1000	1127	1.1

of Observations (*n*) – Test Decision

Table 2: Chi-square distribution test limits and standard deviation scaling factors at a 0.3% significance level

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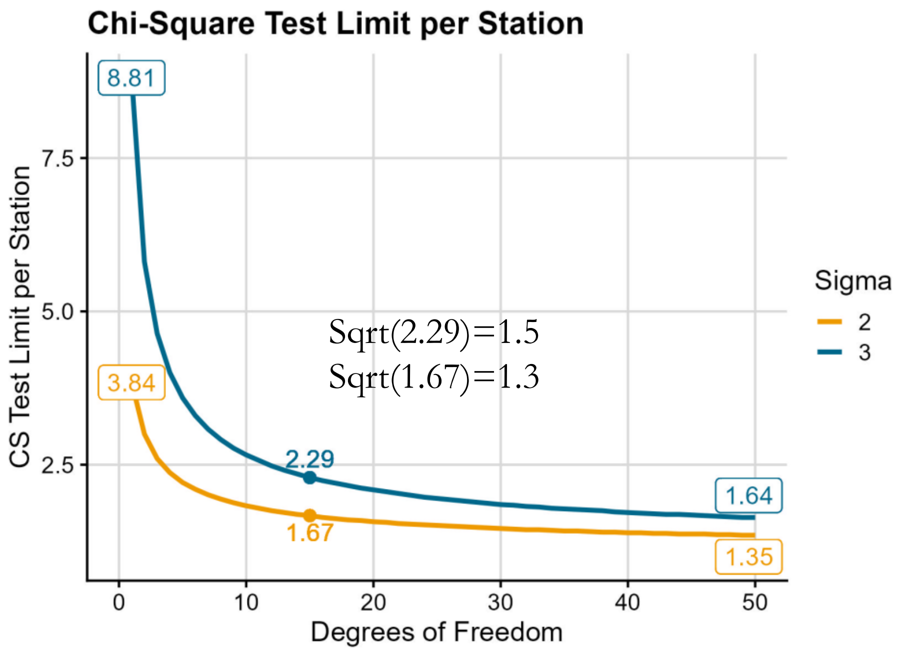
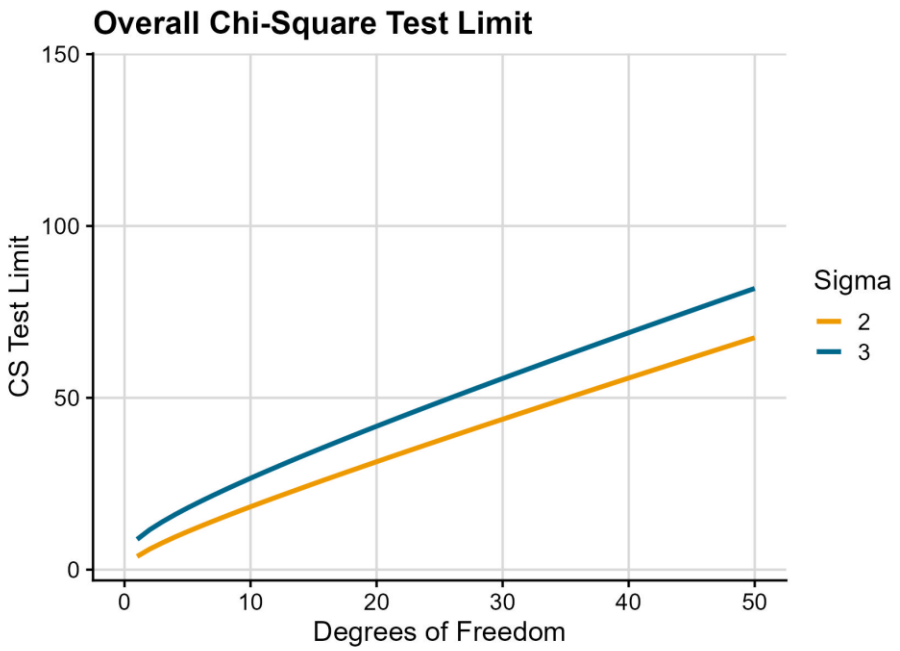
15	34.4	1.5
100	143	1.2
1000	1127	1.1

The magnitude a systematic inclination error must exceed, before a χ_I based Chi-Square test fails a survey, decreases as *n* increases (Ekseth, pg 6).

Too few stations = excessively high sensitivity to random errors

Too many stations = excessively high sensitivity to systematic errors





Unanswered Questions

1. ~~What is our expected uncertainty(standard deviation) to calculate the Chi Square Test values?~~ **Uncertainty Expectation is set at 1 sigma**
    a. SPE-105558 leaves this definition ambiguous
    b. Can expectations set for RIP Test or Ellipse Overlap guide us?
        a. Should these tests reject survey data at similar thresholds?
    c. Is there an ISCWSA SC interested in explicitly defining these QC Tests?

2. ~~If the Chi Square Test Values are a constant in both one-sided and two-sided tests,~~ **why are we making it easier to exceed a Lower Test Limit calculated at a 2 sigma CI (*Z*≈25 vs 34 with n=15)?**
    a. Does SPE-105558 appropriately define IDT/ADT/CODT Chi-Square tests?
    b. Is anyone in our industry running these QC Tests? **A few said they are, but did not provide specifics**

3. How might these QC Tests be applied to a combined survey set?

Concerns

1. Lack of published papers/research in our industry?
   - Two SPE papers by Gyrodata on Chi-Square without the detail required to reproduce results
   - If we can't agree on fundamental aspects on this test, how can we improve it or adjust parameters(ie, point selection for n)?

2. It seems like there is a large gap between when a systematic error would cause each different type of test to fail
   - Chi-Square Tests will fail the earliest so does that mean a large portion of our industry is referencing tests that produce False Positives(ie, optimistic)?

Summary: Chi-Square Test Items to Address

- **Explicitly *define* sigma/scaled variance**
  - What is our expected uncertainty?
- **Is *n* selection appropriate at 15 stations for CODT?**
  - Prone to Type 1 error relative to RIP Mean and Ellipse Test Limits
  - Would n=5 make more sense for CODT?
- 0.003 significance *or* 3 sigma?
- Should we *switch* to the term *"Discrepancy" to* refer to "*measurement differences*"?
- How to run the CODT on a lower Survey Leg?
  - Zero Error Tie in and start ~500' out f/ TIP to avoid small error sensitivity
- 0.1 **or** 0.05 **or** 2 sigma for 2 sided test?
  - Mistake made in paper or appendix?

Table 2: Chi-square distribution test limits and standard deviation scaling factors at a 0.3% significance level

n	$Z_{0.003,n}$	$\sqrt{Z_{0.003,n}/n}$
1	8.8	3.0

Table 6.2 Confidence limits associated with various $\Delta\chi^2$ contours for one degree of freedom.

$\Delta\chi^2$ contour	1.00	2.71	4.00	6.63	9.00
Measurements within range	68.3% 1σ	90.0%	95.4% 2σ	99.0%	99.7% 3σ

Hughes and Hase, Measurements and their Uncertainties – A Practical Guide to Modern Error Analysis

Chi Squared References

SPE/IADC 105558

High-Integrity Wellbore Surveys: Methods for Eliminating Gross Errors

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Published: February 20, 2007 (Peer Reviewed)

IADC/SPE-199554-MS

Validation of Error Models – A Key Component of Risk Mitigation in Wellbore Collision Challenges

Tarig Ali, Adrián Ledroz, and John Weston, Gyrodata; William Allen, BP

Published: February 25, 2020 (Peer Reviewed)

Proposal

1. I would like to see my implementation of Chi-Square per SPE-105558 validated(IDT/ADT/CODT) with a test set of data posted on the ISCWSA website

2. If I summarized my understanding of the Chi-Square test, would there be interest at this SC to publish a communication summarizing all of the test decisions and update the eBook verbiage
   - eBook verbiage on this appears to be roughly the same content as seen in SPE-105558
   - Should this content be added to RP-78 at some point? Currently, the RIP test is only defined in that document and its noted that it is no replacement for Chi-Square

Questions?

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Comparison of Chi-Square Equations

SPE-105558 Chi Square

- Mean expectation is assumed to be zero – Conflicts w/ RIP Test?
- Suggests Variance is required
- Does Gamma selection tell us what magnitude of Power/Probability we can confidently reject surveys?

$$X = \sum_{i=1}^{n} \frac{x_i^2}{\sigma_i^2} \leq Z_{\gamma,n}$$

where $Z_{\gamma,n}$ is the Chi-square test limit for n

General Form of Chi-Square

- "Expectation" is very subjective!

DEFINITION OF CHI SQUARED

If we make n measurements for which we know, or can calculate, th values and the standard deviations, then we define χ^2 as

$$\chi^2 = \sum_{1}^{n} \left(\frac{\text{observed value} - \text{expected value}}{\text{standard deviation}} \right)^2 . \quad [S$$

In the experiments considered in this chapter, the n measurement numbers, $O_1, \ldots, O_n$, of times that the value of some quantity x was c each of n bins. In this case, the expected number E_k is determined by th distribution of x, and the standard deviation is just $\sqrt{E_k}$; therefore,

$$\chi^2 = \sum_{k=1}^{n} \frac{(O_k - E_k)^2}{E_k} . \quad [$$

If the assumed distribution of x is correct, then χ^2 should be of order n the assumed distribution is probably incorrect.

Taylor, Intro to Error Analysis – The Study of Uncertainties in Physical Measurements

Two Routes available for Chi-Square Conclusion?

Binary Pass/Fail Hypothesis Test

X_I	102
Test limit	34
Test conclusion	Fail

The test fails. A failed IDT is either caused by a relative depth difference, or by at least one survey performing outside its error model. Therefore, it is not possible to conclude that something is wrong with the inclination measurements before the relative depth corrections have been checked, and found to be acceptable.

X_I	16.0
Test limit	34
Test conclusion	Pass

The gyro and MWD inclination standard deviations (error model estimates) used are not very different from each other, and the pass conclusion can therefore be taken as evidence that gross inclination errors are not present in either survey.

Every sum of Xi calculated has an associated power/probability(p value[x-axis below]) that we can reject the distribution

- For the IDT example to the right with a Pass result, there is a ~40% chance of seeing this discrepancy or larger with our expected uncertainty distribution?

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58

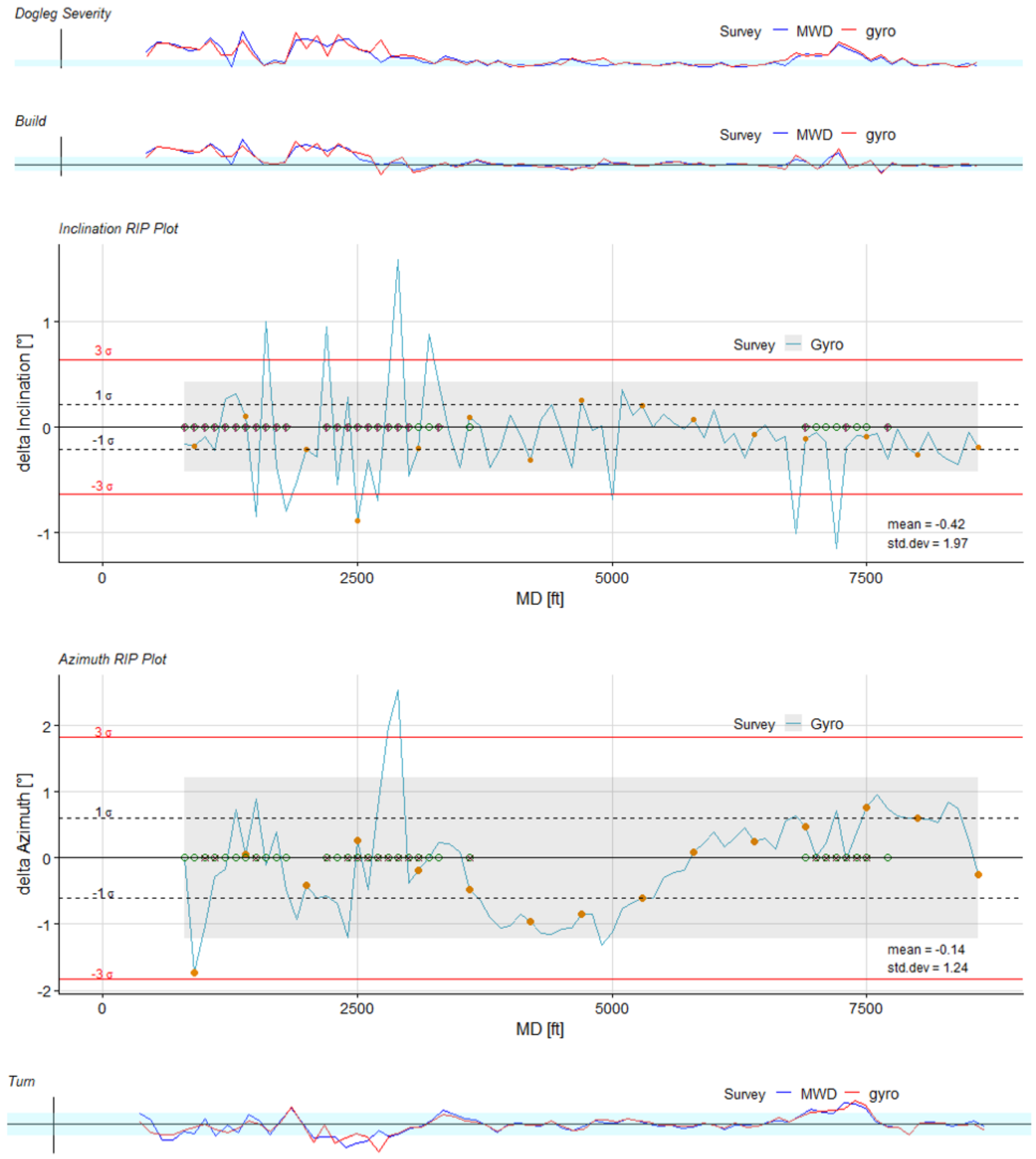
https://passel2.unl.edu/view/lesson/9beaa382bf7e/8

Survey Selection after n is determined – Test Decision

1. Should we stick with a consistent decision of equally spacing out the surveys selected or should we adjust selections based off of our knowledge of the surveying acquisition process and environment
   - Should we ensure areas of high and low DLS are sampled to some predetermined ratio
   - Should 3 sigma outliers be rejected from this test if in a high dogleg area?

2. How should the interpolation process work?
   - If CLs are significantly different, it doesn't seem fair to penalize the lower frequency survey even though it may not be the an accurate representation of the wellbore between stations

RIP Test with Dogleg Severity, Build, and Turn



Shaded areas:

grey = 2 sigma tolerance level

light blue = dogleg severity: between 0 and 1, turn and build: between -1.5 and 1.5

Well Stations:

orange point = Chi-Square test, green circle = high dogleg severity, red cross = high build, pink X = high turn

IDT/ADT/CODT Equations

IDT Example:

- ADT/CODT equations are similar
- Variance Scaled:
  - Standard Deviation = 1 Sigma Std Dev f/EM, 2 sigma, or 3 sigma
- Inclination Difference = **Inclination Discrepancy**
  - Bevington definition is a differences in repeated measurements that arise because we can only determine a result to a given uncertainty

The Chi-square distribution statistical test. A Normally distributed measurement (x) with zero expectation and variance, σ^2, is transformed into an apparent one degree of freedom Chi-square distributed measurement by squaring the measurement and dividing by the variance. A given number

$$X = \sum_{i=1}^{n} \frac{x_i^2}{\sigma_i^2} \le Z_{\gamma,n}$$

By assuming that the two overlapping inclination measurements at the same depth are uncorrelated and Normally distributed, the following random variable (x_I) can be formed:

$$x_I = \frac{\Delta I^2}{\sigma_{I1}^2 + \sigma_{I2}^2} \qquad (5)$$

where ΔI is the inclination difference between the two independent surveys at a given depth, and σ_{I1} and σ_{I2} are the inclination standard deviations of the first and second surveys respectively. σ_{I1} and σ_{I2} can be calculated with the help of the error model input values and weighting functions.

Appendix 1C: Equations for the inclination difference test

Inclination difference at station (depth) i:

$$\Delta I_i = I2_i - I1_i$$

Variance scaled squared inclination difference at station (depth) i:

$$x_{I,i} = \frac{\Delta I_i^2}{\sigma_{I1,i}^2 + \sigma_{I2,i}^2}$$

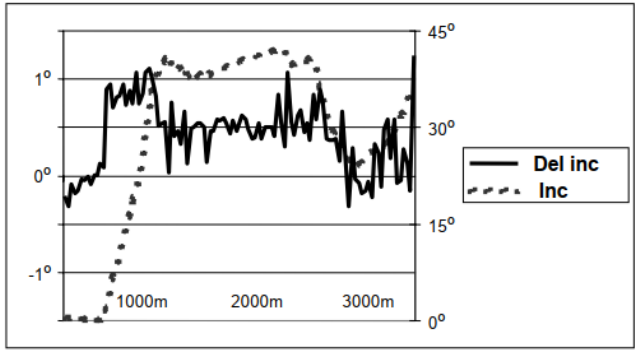
Test variable:

$$X_I = \sum_{i=1}^{n} x_{I,i}$$

where n is the total number of stations used.

Pass/Fail IDT Examples

- Failure Statement from First IDT to the Left is a much stronger statement
- Statistics can only disprove things
- SPE-77221 statement that Ellipse overlap confirms surveys – conflicts w/ Chi-Square Test

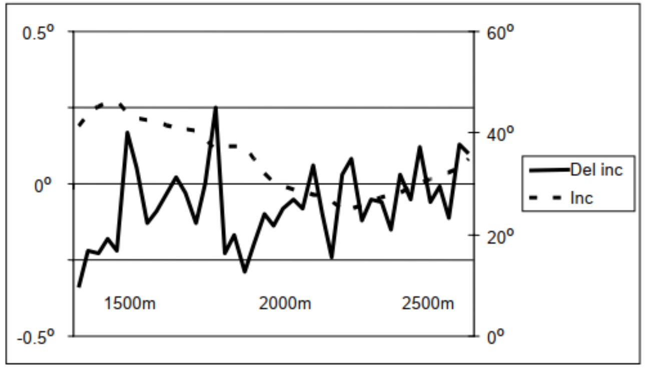


A 15 station IDT was run to find out if these inclination differences are acceptable relative to the tool error models assigned to the surveys. The MWD survey was assigned to the SPE-WPTS sag corrected MWD error model, and the gyro survey to the stationary error model recommended by the gyro service provider. The following test result was achieved:

X_I	102
Test limit	34
Test conclusion	Fail

The test fails. A failed IDT is either caused by a relative depth difference, or by at least one survey performing outside its error model. Therefore, it is not possible to conclude that something is wrong with the inclination measurements before the relative depth corrections have been checked, and found to be acceptable.

A second IDT example is shown in the following figure. It is based on inclination differences between a continuous gyro survey and a MWD section of 1300 metres length.



The 15 station inclination difference test gave the following result for this second example:

X_I	16.0
Test limit	34
Test conclusion	Pass

The gyro and MWD inclination standard deviations (error model estimates) used are not very different from each other, and the pass conclusion can therefore be taken as evidence that gross inclination errors are not present in either survey.

Survey Uncertainty Quantification with R: The Combined Survey Method

Mike Calkins – Three Sigma Well Design, LLC

Overview

1. ***Why?***
2. Combined Survey Methodology
   a) *SLB Patent*
   b) *ISCWSA eBook*
3. Common Survey QC Tests
   a. Qualitative Ellipse Visual Tests
   b. RIP Test
   c. Chi-Squared Tests
      1. One Sided for Individual Wells
      2. Two Sided for EM Validation & Refinement
4. Current Chi-Square Test Implementation per Ekseth *et al.*, 2007 (SPE-105558)
   a) Limitations, Assumptions, & Concerns
   b) Need to explicitly define all QC Tests so they can be run correctly and consistently
5. Overview of R and preview of current QC Report code

Why?

1. To **explicitly define uncertainty expectations** for survey data and the **means to determine** when a tool is not performing as assumed by the EMs
    - **ISCWSA OWSG Mission Statement**: To promote practices that provide confidence that reported positions are within their stated uncertainty

2. "To **obtain the maximum amount of useful information from the data on hand** without being able to repeat the experiment with better equipment or reduce statistical uncertainty by making more measurements"

    - Bevington, Data Reduction and Error Analysis for the Physical Sciences

Expired SLB Patent

- Described as the "Most Accurate Position"
- Caveat that one survey type must be a wireline survey.
  - In-Line with SPE-105558 guidance to ensure errors are not correlated
- Neither Patent or SPE papers elaborate on limitations of Reducing Uncertainty
  - Fig 5 visual suggests Uncertainty can be reduced ~50%
- First Commercial Application only occurred recently per SLB Paper and ISCWSA Presentation
  - Implementation Challenges and Demand from Operators caused this to stagnate?

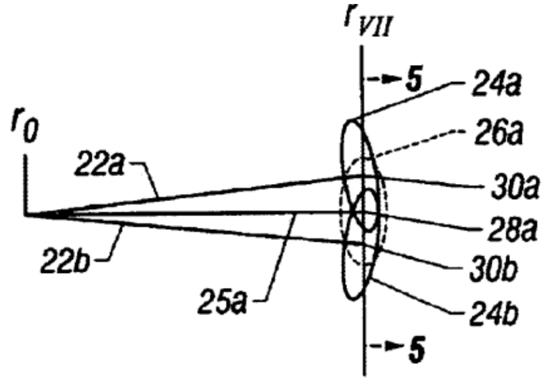


FIG. 4

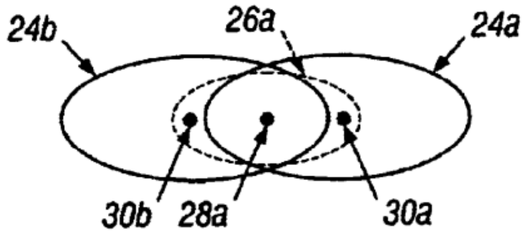


FIG. 5

US006736221B2

(12) **United States Patent**
Chia et al.

(10) **Patent No.: US 6,736,221 B2**
(45) **Date of Patent: May 18, 2004**

(54) **METHOD FOR ESTIMATING A POSITION OF A WELLBORE**

(75) Inventors: **Christopher R. Chia**, Houston, TX (US); **Wayne J. Phillips**, Houston, TX (US); **Darren Lee Aklestad**, Sugar Land, TX (US)

(73) Assignee: **Schlumberger Technology Corporation**, Sugar Land, TX (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

(21) Appl. No.: **10/028,864**

(22) Filed: **Dec. 21, 2001**

(65) **Prior Publication Data**

US 2003/0121657 A1 Jul. 3, 2003

(51) **Int. Cl.**[7] **E21B 47/00**
(52) **U.S. Cl.** **175/45**; 33/303
(58) **Field of Search** 175/45; 73/152.03; 33/302, 303, 304, 313; 702/6, 9, 10

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Search Report Under Section 17 dated Mar. 26, 2003 for GB0224249.3.

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Primary Examiner—David Bagnell
Assistant Examiner—Matthew J Smith
(74) *Attorney, Agent, or Firm*—J.L. Jennie Salazar; Brigitte L. Jeffery; John Ryberg

(57) **ABSTRACT**

A method is disclosed which utilizes multiple overlapping surveys to estimate a position in a wellbore and related position uncertainty. Multiple surveys are often taken over the same portion of a wellbore either concurrently or sequentially and/or using various instruments. Each survey generates an estimated survey position and related uncertainty for a given location in the wellbore. By combining the estimated survey positions and uncertainties for these overlapping surveys, a resultant position and related ellipsoid of uncertainty is estimated. This resultant position estimates a position in the wellbore by incorporating the estimated survey positions and uncertainties of multiple overlapping surveys.

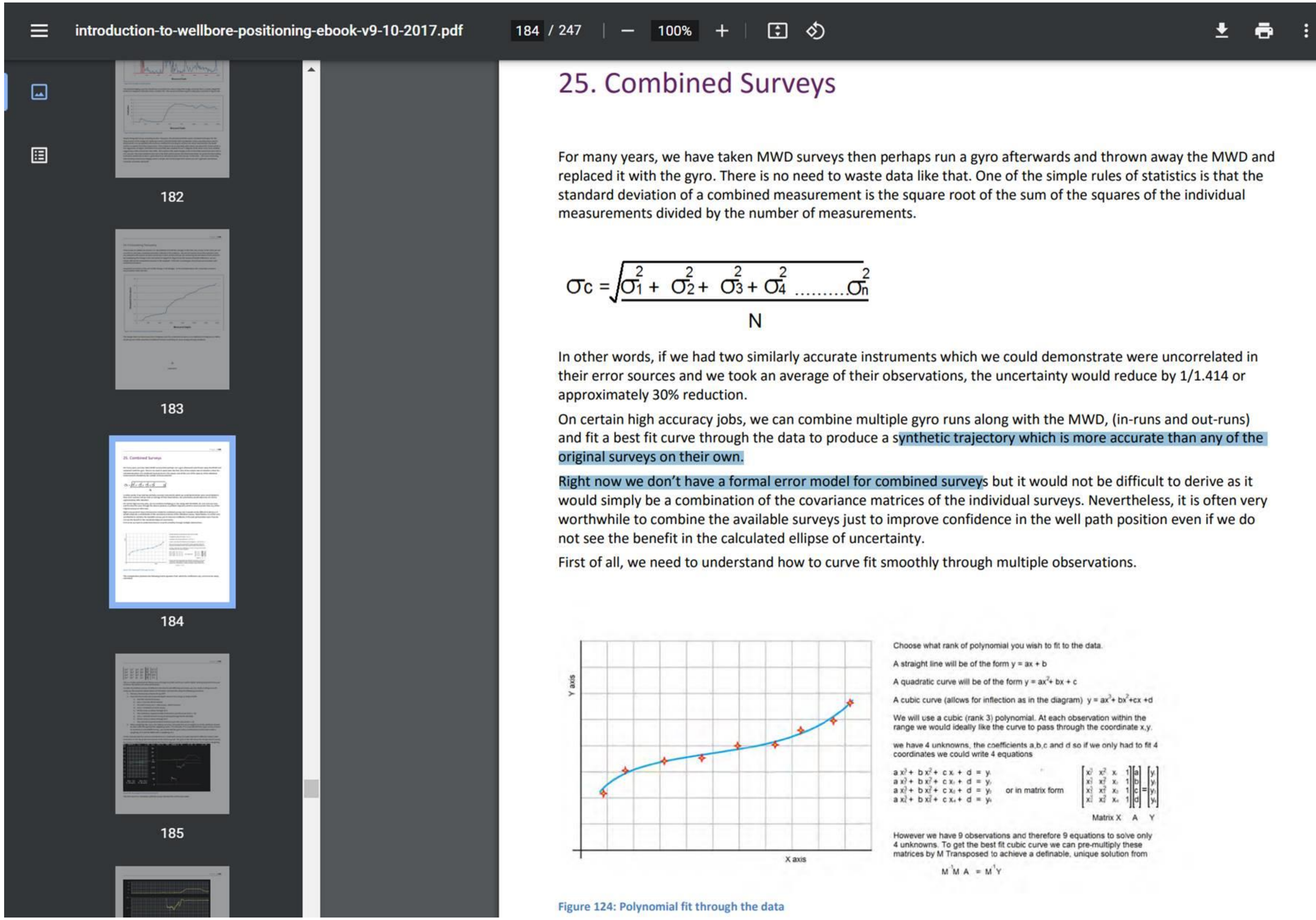
26 Claims, 3 Drawing Sheets

ISCWSA eBook

- **Uncertainty** of combined measurements **reduces by a factor of 1/sqrt(n) if survey uncertainty in each tool is equivalent**
  - 2 surveys - 1/sqrt(2)= 0.71
  - 3 surveys - 1/sqrt(3)=0.58
  - 5 Surveys - 1/sqrt(5)=0.45
  - 50 Surveys - 1/sqrt(50)=0.14
- What exactly is this simple rule in statistics and what assumptions are being made?
  - Simple Rules that Errors add in quadrature (i.e., RSS/Euclidean Norm) and Standard Error (SE - SDOM)

$$SE = \frac{\sigma}{\sqrt{n}}$$

  - ***More context is needed in the eBook!***
- ***If we are taking the standard deviation of these overlapping measurements, the mean is assumed to be zero and we divide by n instead of n-1***
  - ***Large discrepancies are likely telling us this is a bad assumption***
- At what point should Uncertainty not be reduced further?
  - Goodness-of-Fit Testing of a null hypothesis via Chi Square to make this decision?
  - If Chi-Square Disproves our uncertainty expectation, we should not use this



25. Combined Surveys

For many years, we have taken MWD surveys then perhaps run a gyro afterwards and thrown away the MWD and replaced it with the gyro. There is no need to waste data like that. One of the simple rules of statistics is that the standard deviation of a combined measurement is the square root of the sum of the squares of the individual measurements divided by the number of measurements.

$$\sigma_c = \frac{\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 \ldots\ldots\ldots \sigma_n^2}}{N}$$

In other words, if we had two similarly accurate instruments which we could demonstrate were uncorrelated in their error sources and we took an average of their observations, the uncertainty would reduce by 1/1.414 or approximately 30% reduction.

On certain high accuracy jobs, we can combine multiple gyro runs along with the MWD, (in-runs and out-runs) and fit a best fit curve through the data to produce a synthetic trajectory which is more accurate than any of the original surveys on their own.

Right now we don't have a formal error model for combined surveys but it would not be difficult to derive as it would simply be a combination of the covariance matrices of the individual surveys. Nevertheless, it is often very worthwhile to combine the available surveys just to improve confidence in the well path position even if we do not see the benefit in the calculated ellipse of uncertainty.

First of all, we need to understand how to curve fit smoothly through multiple observations.

Figure 124: Polynomial fit through the data

"there comes a point at which further knowledge is unobtainable"

– Bevington, Data Reduction and Error Analysis for the Physical Sciences

Systematic Errors(eBook cont.) – Enemy #1?

1. Important assumption is that all systematic errors have been removed for this "simple rule" to be true?
   - eBook only suggests that surveys must have uncorrelated errors for this to work
   - Should we assume large discrepancies must be caused by a notable systematic error?

2. Is the statistics Definition of Systematic Error the same as ours(see below) or is the focus on Bias? Are these terms used interchangeably like Uncertainty and Error?
   - Statistics terminology is confusing!

3.3 Systematic Errors

Systematic errors are all remaining errors when gross- and random- errors are removed. A systematic error has the same size, sign or geometric dependent nature for a given number of measurements. This means that some gross errors, like for instance use of wrong calibration constants, are systematic errors for all measurements of a given type. It is,

– Ekseth, Uncertainties in Connection with the Determination of Wellbore Positions (March 1998, PHD Dissertation – Foundational Document referenced by ISCWSA)

SPE-85111/77221: Papers on SLB "MAP" Process

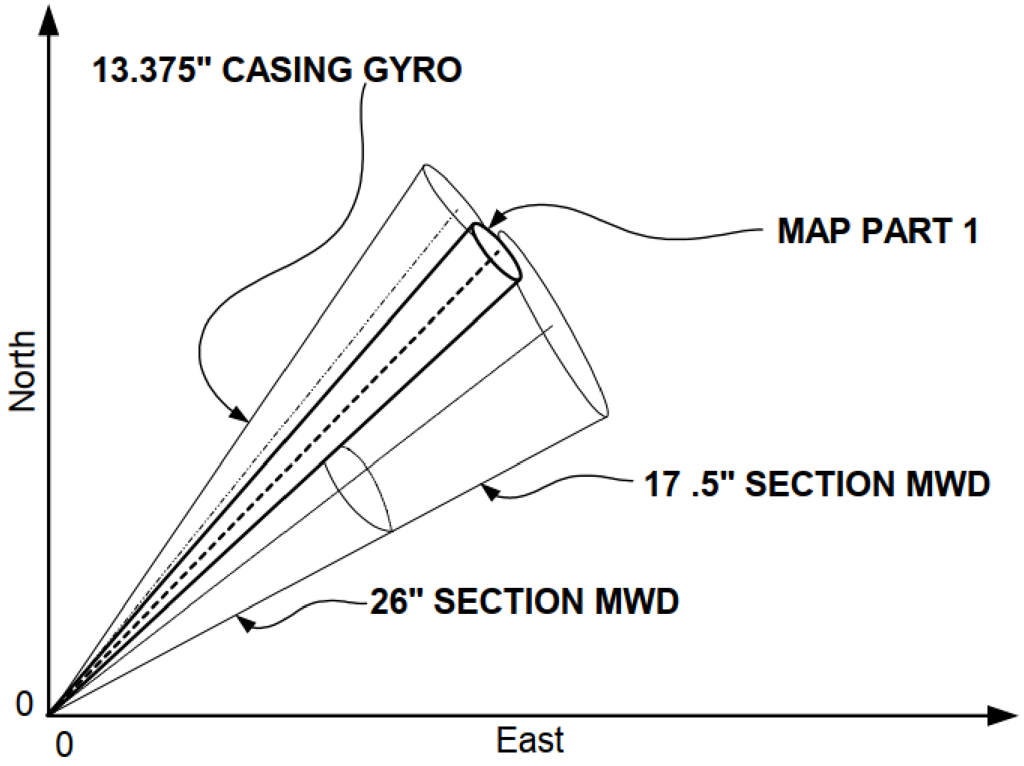


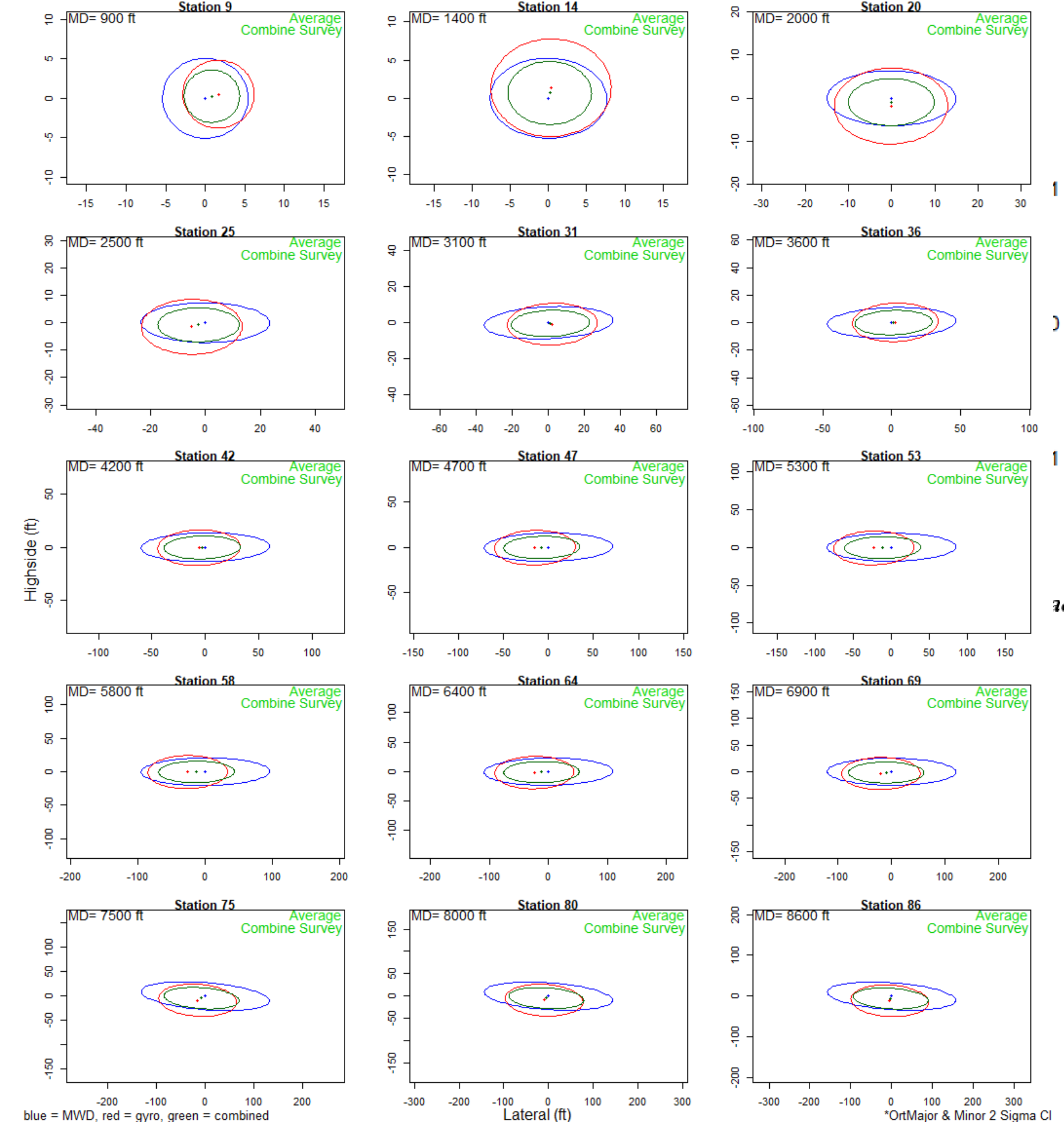
Fig. 4 - Schematic for the same planned directional well, for the first series of drilling surveys and the first casing gyro survey, where the MAP technique is used to obtain a more accurate updated wellpath position, from which drilling progress will continue.

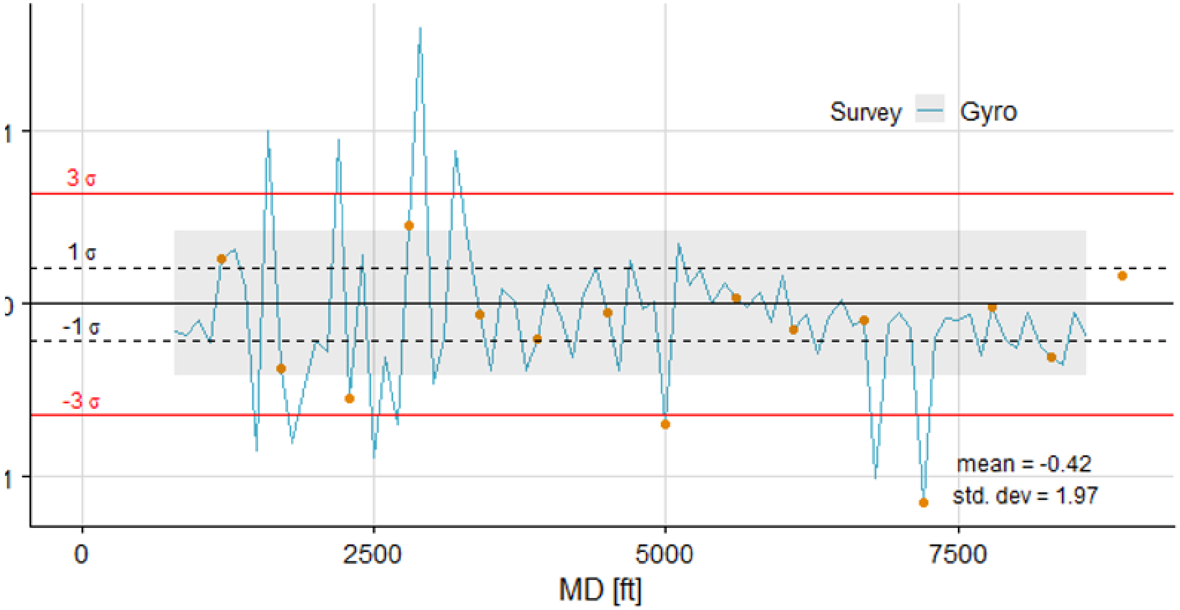
first two drilling stages (26" hole section and 17.5" hole section) are completed prior to a casing gyro being run in the surface casing string. Typically, the resultant gyro survey could show a significant shift in well position from the drilling surveys, but provided the EOU's at least touched, then statistically this would be accepted as having been confirmed by the MWD, and the gyro would then be used to update the current well location prior to drilling ahead with the 12.25" hole section. The next hole section drilling surveys are then

- Reword the statement above to be in-line with SPE-105558?
  - With data available we can't disprove either EM
  - Only evidence that the observations made did not disprove the EMs referenced
- CI of surveys is not mentioned
- Good Survey/Poor Survey Classification?

Thoughts on Limitations

1. The two surveys in question can not be more than 40% more accurate than the other
   - SLB SPE Papers suggest this is not a limitation(Fig 4 on Prev Slide)
2. For an individual survey tool, this logic can only reduce random errors.
   - We are stuck with whatever systematic errors are present
3. We should assume large HLA/NEV coordinate discrepancies are caused by Systematic Errors
4. Chi-Square QC Test may not be defined appropriately (ie, it seems like it fails too easily)
   - QA/QC SC input?





aded area = Tolerance, orange dots = 15 stations used for the Chi-Square Test

Table 2: Result of all Chi Square (X^2) tests

	X^2 Test Value	Test Limit	Test Conclusion
IDT	60.19	34.4	Fail
ADT	27.88	34.4	Pass
CODT (HLA)	-	-	Pass
X_L	1.01	34.4	Pass
X_H	0.97	34.4	Pass
X_W	0.29	34.4	Pass

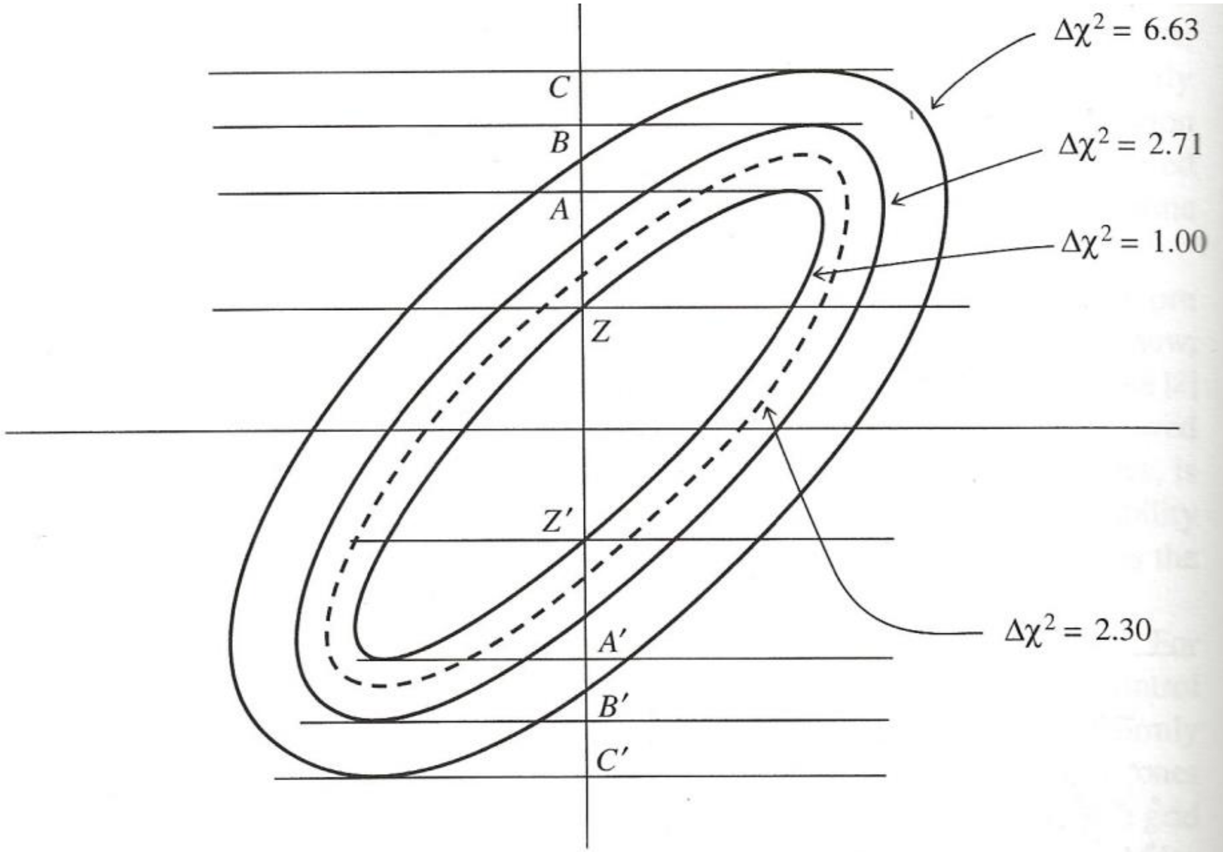


Figure 15.6.4. Confidence region ellipses corresponding to values of chi-square larger than the fitted minimum. The solid curves, with $\Delta\chi^2 = 1.00, 2.71, 6.63$ project onto one-dimensional intervals AA', BB', CC'. These intervals — not the ellipses themselves — contain 68.3%, 90%, and 99% of normally distributed data. The ellipse that contains 68.3% of normally distributed data is shown dashed, and has $\Delta\chi^2 = 2.30$. For additional numerical values, see accompanying table.

Press, W. H., B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes, The Art of Scientific Computing,* Cambridge University Press, New York (1986).

Questions?

Weighted Average

Follows the Principle of Maximum Likelihood

The solution of this equation for X is our best estimate and is easily seen to be

$$\text{(best estimate for } X) = \left(\frac{x_A}{\sigma_A^2} + \frac{x_B}{\sigma_B^2}\right) \Big/ \left(\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2}\right). \qquad (7.7)$$

This rather ugly result can be made tidier if we define **weights**

$$w_A = \frac{1}{\sigma_A^2} \quad \text{and} \quad w_B = \frac{1}{\sigma_B^2}. \qquad (7.8)$$

With this notation, we can rewrite (7.7) as the **weighted average** (denoted x_{wav})

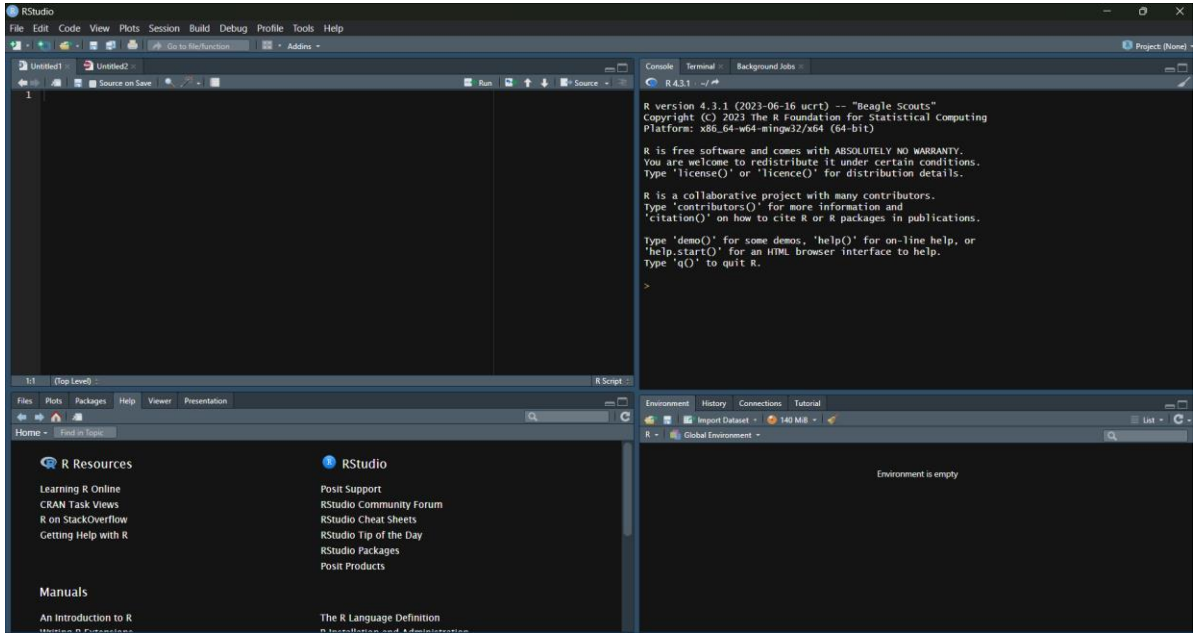
$$\text{(best estimate for } X) = x_{\text{wav}} = \frac{w_A x_A + w_B x_B}{w_A + w_B}. \qquad (7.9)$$

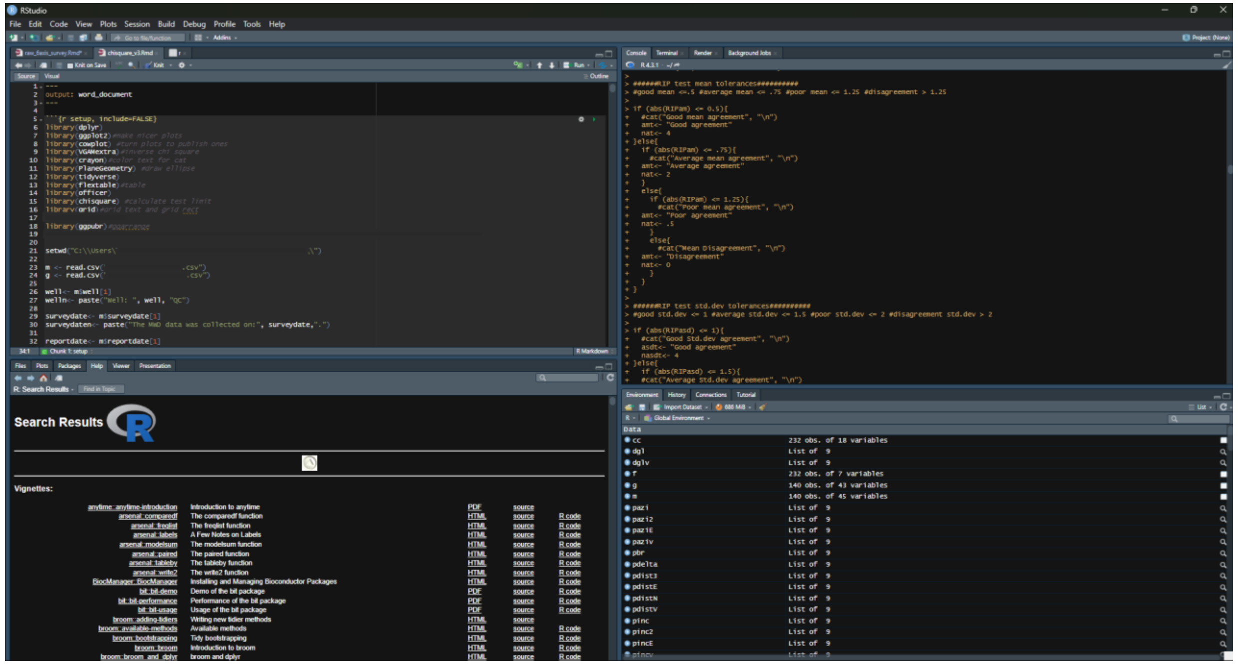
If the original two measurements are equally uncertain ($\sigma_A = \sigma_B$ and hence $w_A = w_B$), this answer reduces to the simple average $(x_A + x_B)/2$. In general, when $w_A \neq w_B$, the weighted average (7.9) is *not* the same as the ordinary average; it is similar to the formula for the center of gravity of two bodies, where w_A and w_B are the actual weights of the two bodies, and x_A and x_B their positions. In (7.9), the "weights" are the inverse squares of the uncertainties in the original measurements, as in (7.8). If A's measurement is more precise than B's, then $\sigma_A < \sigma_B$ and hence $w_A > w_B$, so the best estimate x_{best} is closer to x_A than to x_B, just as it should be.

R Statistical Programing

What is R?

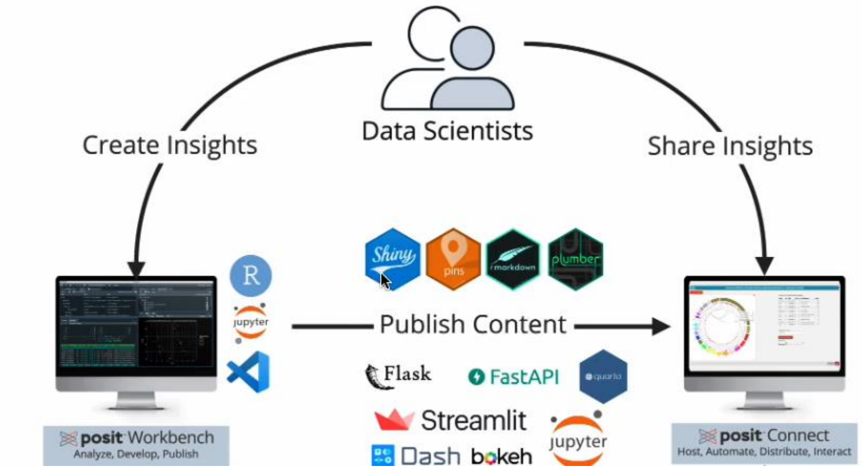
- An open-source statistical computing and graphic coding program
- Handles and stores data
- Computes large data and operations
- Functions not available in base package can be easily added by importing other created packages, or you can create your own functions.
- Most users use R studio as it is a more user-friendly interface than R.





R Studio Support

- While R Studio is a free program, they do offer consulting support for a fee
- Used by some fortune 100 companies
- Access to tools/packages with license
- Able to connect to remote sessions

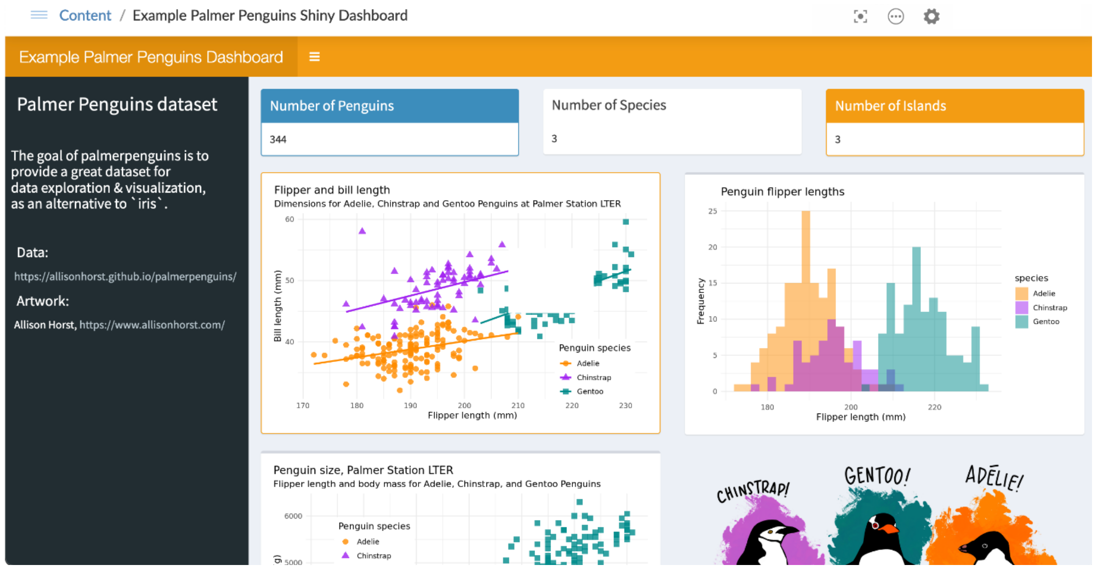


janssen PHARMACEUTICAL COMPANIES OF Johnson & Johnson
mastercard.
aetna
NASA
Walmart

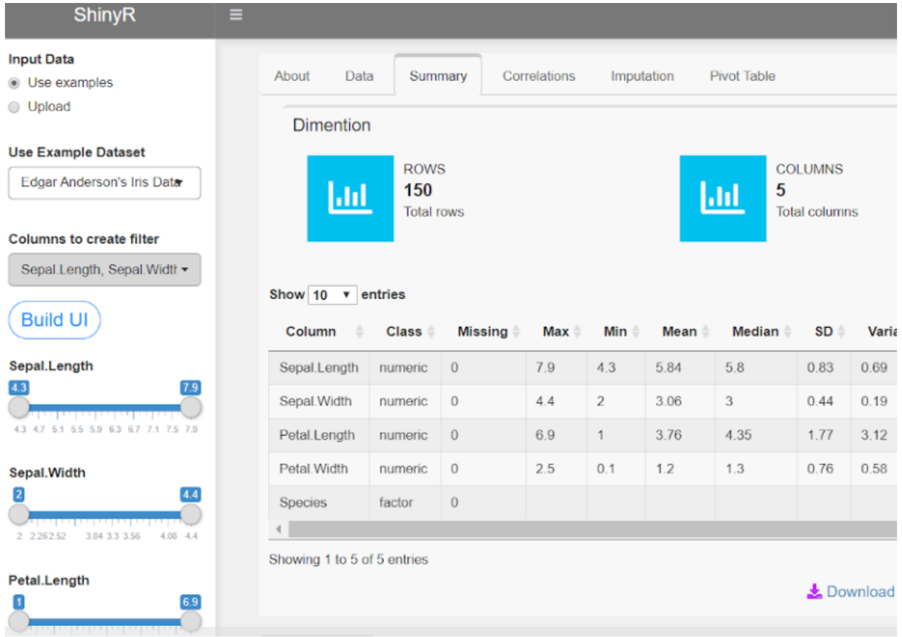
Fortune 100 companies that use R studio

R Shiny

- Extension of R
- App like; more interactive than document
- Need a dashboard to share to others
  - Dashboard can be free or paid services
  - Examples: Open-Source Shiny Server, Posit Connect, shinyapps.io, shinyproxy
  - Can be password protected
- Dashboards are customizable to the coder and depends on the server



Example of Posit Connect Dashboard



Example of Open-Source Shiny Server

Current QC Report Creation Process

- Export survey data by survey leg from directional software
  - Minor cleanup via python required for easy R import
- Point to survey data file name in Code
- Manually enter a rough INC/AZI error in degrees due to software export limitation
  - HAL and SLB reps at OWSG meeting said they are working to add this functionality
  - This is manageable since COMPASS will export NEV/HLA uncertainty values required for the CODT(most import test in my view)
  - IDT & ADT Chi-Square Tests are a bit duplicative since we can RIP test in R and COMPASS
- Run report in pdf, word, or html
- Report contains the following plots and data tables to audit results as needed.
  - Transparency of Calculations has been a key focus area
- Reports have not been tested at scale yet due to Chi-Square Interpretation concerns and INC/AZI Error data export limitation

RIP Test

```
#delta calculation, ref = MWD, offset = gyro
si9$deltaInc <- (si9$Inc.y-si9$Inc.x)

#Inc.x error
si9$ier.xsd<- (0.3*.5)  #(*0.5 = 1 sigma, *1 = 2 sigma, *1.5 =

#Inc.y error
si9$ier.ysd<- (0.3*.5) #(*0.5 = 1 sigma, *1 = 2 sigma, *1.5 =

#Expected error
si9$sd.e.error <- sqrt(si9$ier.xsd^2 + si9$ier.ysd^2)

#normalized inclination differences (std.dev)
si9$normInc.sd <- si9$deltaInc/si9$sd.e.error

#mean (systematic) correct RIP mean
RIPim<-round(mean(si9$normInc.sd),2)

#correct (Random) RIP std.dev
RIPisd<- round(sd(si9$normInc.sd),2)

#tolerance level = mean of expected error * 2
itol<- round((mean(si9$sd.e.error)*2), 2)
```

Delta calculation

MWD & Gyro Error

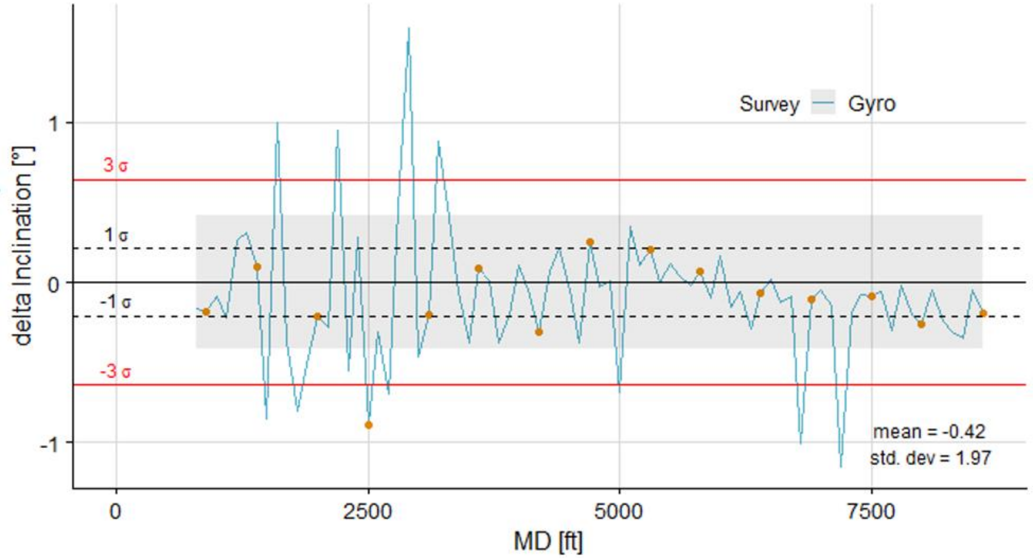
Expected error calculation

Normalized differences

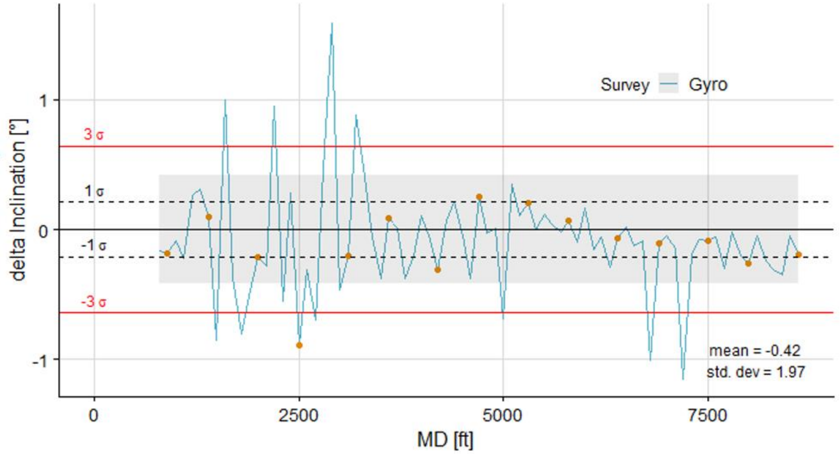
Mean calculation

St.dev calculation

Mean calculation

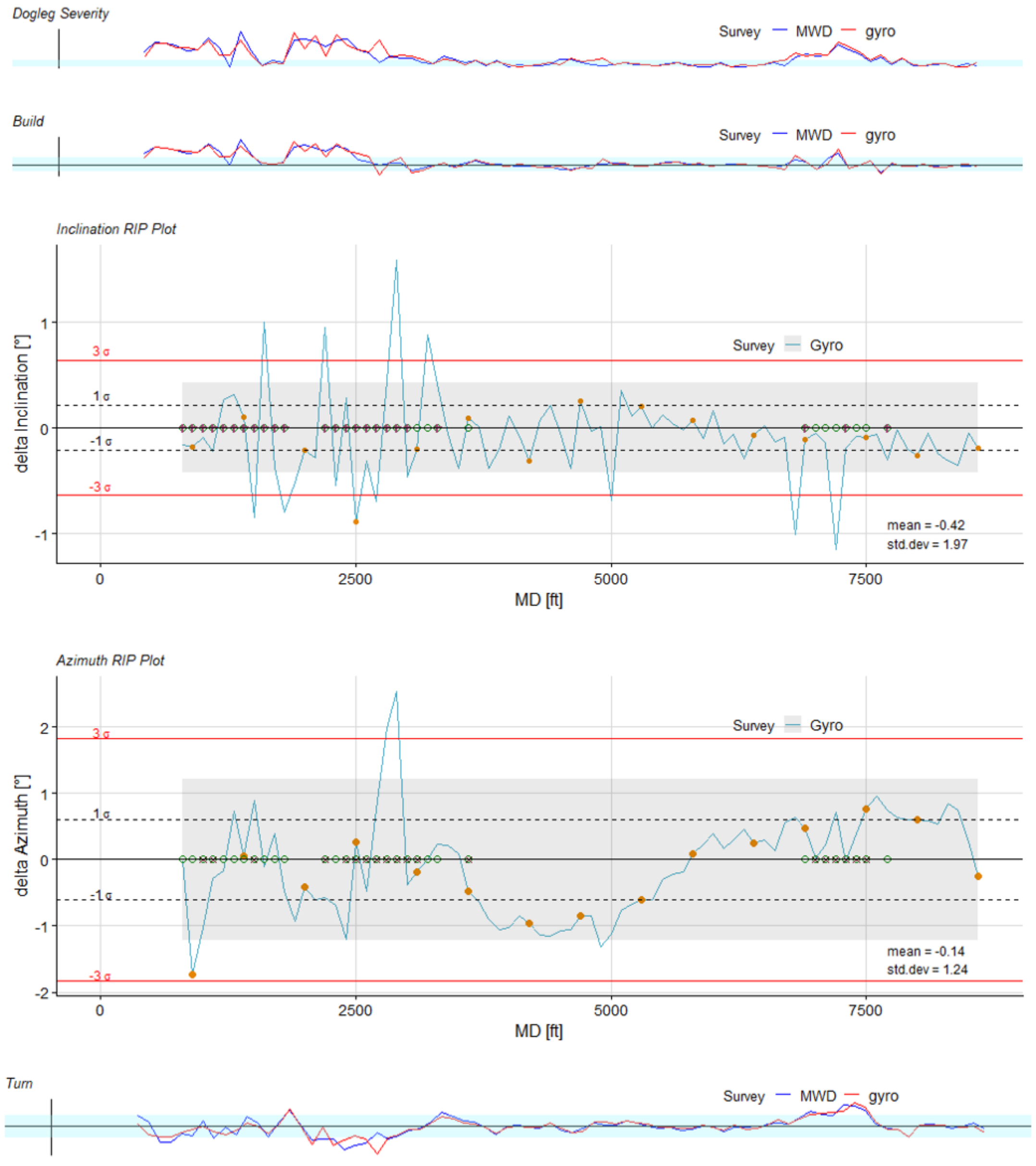


RIP Test




```
pinc <- ggplot(si9, aes(x=MD.x, y=deltaInc, lty = "Gyro")) +
        geom_line(color = linec)+ #delta line
        geom_point(data= si15, aes(x = MD.x, y = deltaInc), color = d15c, size = 1.5)+ #chi-square points
        geom_hline(yintercept = 0)+ #y = 0 line
        geom_ribbon(aes(ymin = 0 - itol, ymax = 0 + itol), alpha = .1)+ #tolerance level
        geom_hline(yintercept = itol3, linetype= "solid", col = "red")+ #tolerance 3 sigma line
        annotate("text", x =2, y = itol3+.1, label= expression("3"~sigma), col = "red", size = 3.25)+ #tolerance 3 sigma text
        geom_hline(yintercept = -itol3, linetype= "solid", col = "red")+ #tolerance -3 sigma line
        annotate("text", x =0, y = -itol3+.1, label= expression("-3"~sigma), col = "red", size = 3.25)+ #tolerance -3 sigma text
        geom_hline(yintercept = itol1, linetype= "dashed", col = "black")+ #tolerance 1 sigma line
        annotate("text", x =2, y = itol1+.1, label= expression("1"~sigma), col = "black", size = 3.25)+ #tolerance 1 sigma text
        geom_hline(yintercept = -itol1, linetype= "dashed", col = "black")+ #tolerance -1 sigma line
        annotate("text", x =2, y = -itol1+.1, label= expression("-1"~sigma), col = "black", size = 3.25)+ #tolerance -1 sigma text
        labs(title = paste('Well:', well),subtitle = paste('Survey leg:', sut), x = "MD [ft]", y = "delta Inclination [°]", linetype = "Survey") + #
labels
        annotate("text", x =8100, y = -1 , label = paste(c("mean =" ,RIPim, "\nstd. dev =", RIPisd ), collapse =  " "),size =3.25)+ #mean & sd labe
        cowplot::theme_cowplot(12)+ #make ggplot more like academic plots
        theme(plot.subtitle=element_text(size=9, face="italic", color="black"))+ #format subtitle
        background_grid()+ #add background grid
        theme(legend.position = c(0.7,.8),legend.title = element_text(size = 10), legend.direction= "horizontal") #place and arrange legend
```

RIP Test with Dogleg Severity, Build, and Turn



Shaded areas:
grey = 2 sigma tolerance level
light blue = dogleg severity: between 0 and 1, turn and build: between -1.5 and 1.5
Well Stations:
orange point = Chi-Square test, green circle = high dogleg severity, red cross = high build, pink X = high turn

Chi-Square Test

```r
#delta of the gyro to MWD
si15$deltaInc <- (si15$Inc.y-si15$Inc.x)

#variance of INC
ivar<- (0.3*.5)^2

#variance scaled
si15$xi <-(si15$deltaInc^2)/((ivar)+(ivar))

#x value for
xi<- round(sum(si15$xi),2)
```

Δx_i^2 (annotation: `si15$deltaInc`)

$\sigma_{1,i}^2$ (annotation: `ivar`)

$\frac{\Delta x_i^2}{(\sigma_{1,i}^2 + \sigma_{2,i}^2)}$ (annotation: `si15$xi`)

$\sum_{i=1}^{n} \frac{\Delta x_i^2}{(\sigma_{1,i}^2 + \sigma_{2,i}^2)}$ (annotation: `xi`)

```r
test <- round(qchisq(p=0.003, df=15, lower.tail=FALSE), 1)
```

$Z_{\gamma,n}$ (annotation: `test`)

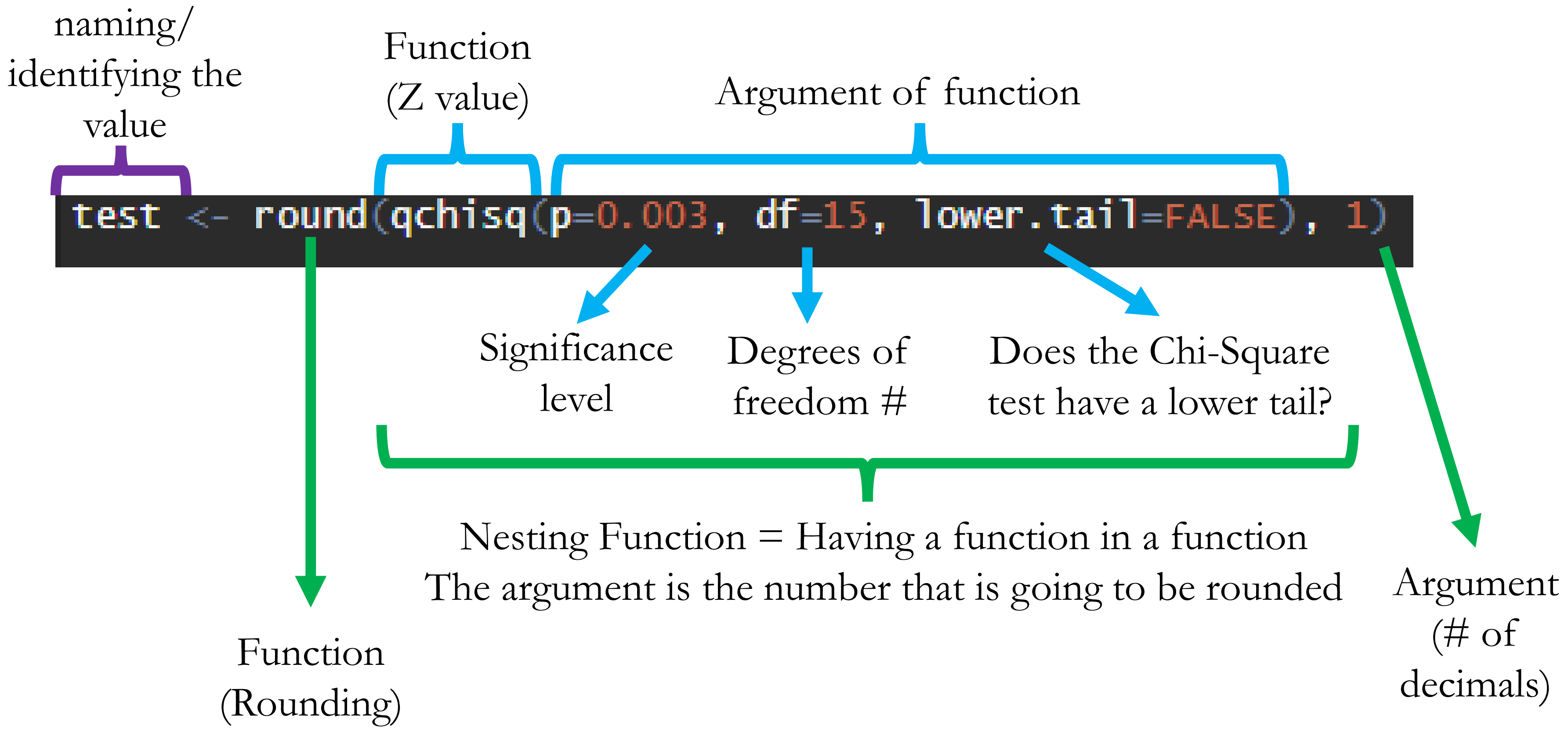
$$\chi = \sum_{i=1}^{n} \frac{\Delta x_i^2}{(\sigma_{1,i}^2 + \sigma_{2,i}^2)} \leq Z_{\gamma,n}$$

x: inclination, azimuth, or CODT (highside/ lateral/ or along-hole)

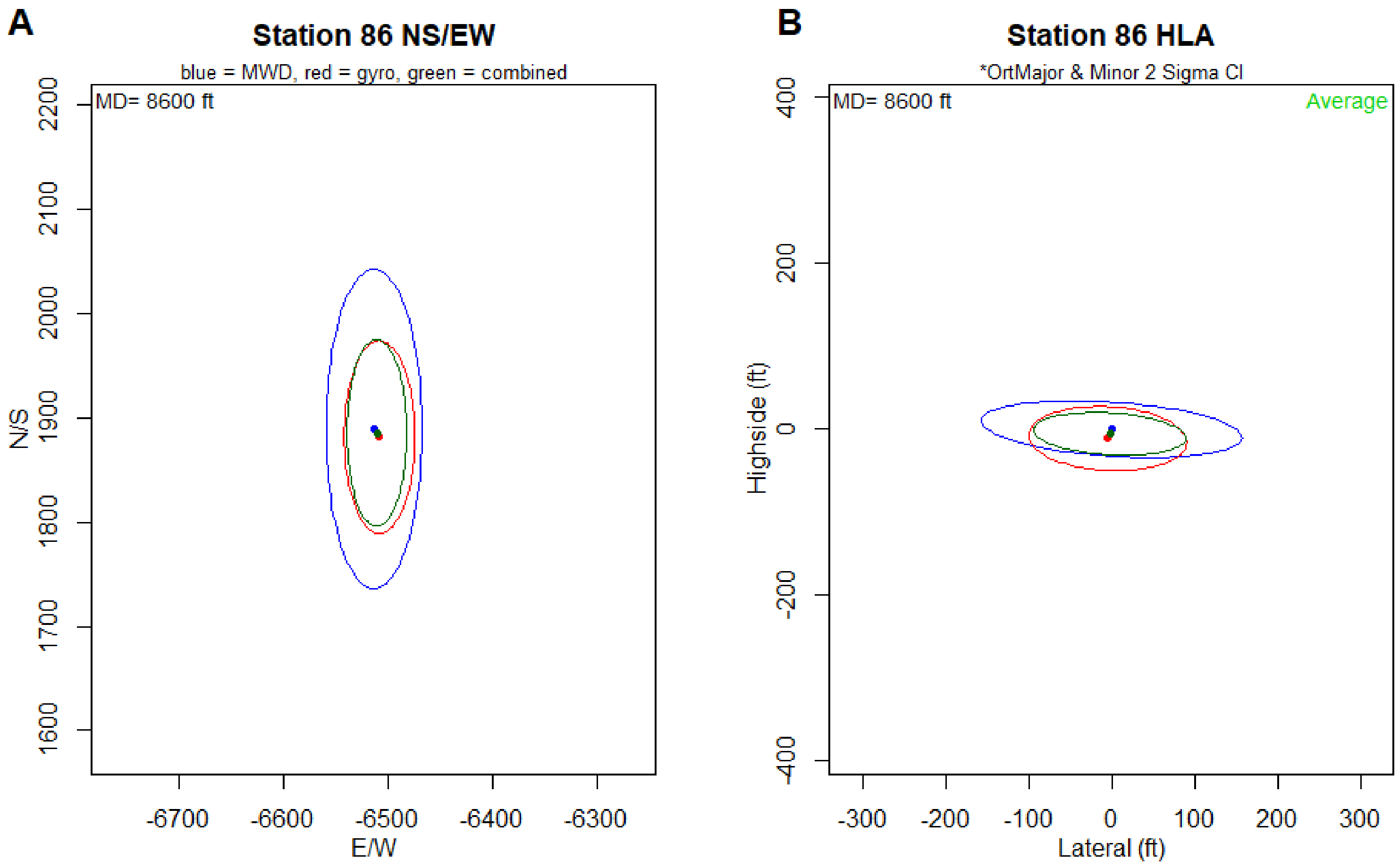
Table 2: Result of all Chi Square (X^2) tests

	X^2 Test Value	Test Limit	Test Conclusion
IDT	27.98	34.4	Pass
ADT	18.39	34.4	Pass
CODT (HLA)	-	-	Pass
X_L	1.23	34.4	Pass
X_H	0.87	34.4	Pass
X_W	0.25	34.4	Pass

R code

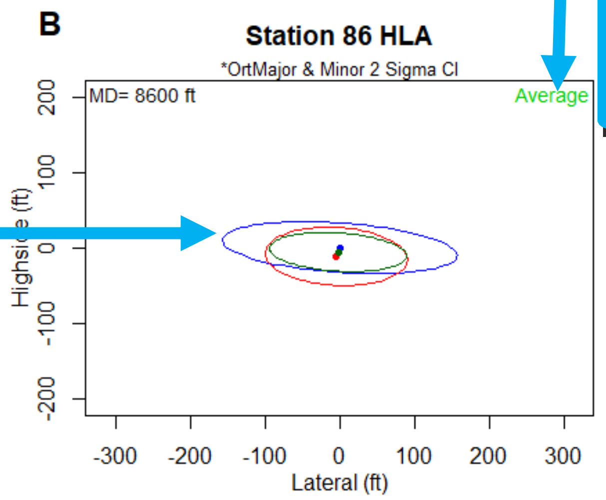

naming/ identifying the value
Function (Z value)
Argument of function
test <- round(qchisq(p=0.003, df=15, lower.tail=FALSE), 1)
Significance level
Degrees of freedom #
Does the Chi-Square test have a lower tail?
Nesting Function = Having a function in a function
The argument is the number that is going to be rounded
Function (Rounding)
Argument (# of decimals)

Ellipse Test with Combined Survey



Ellipse Test Code

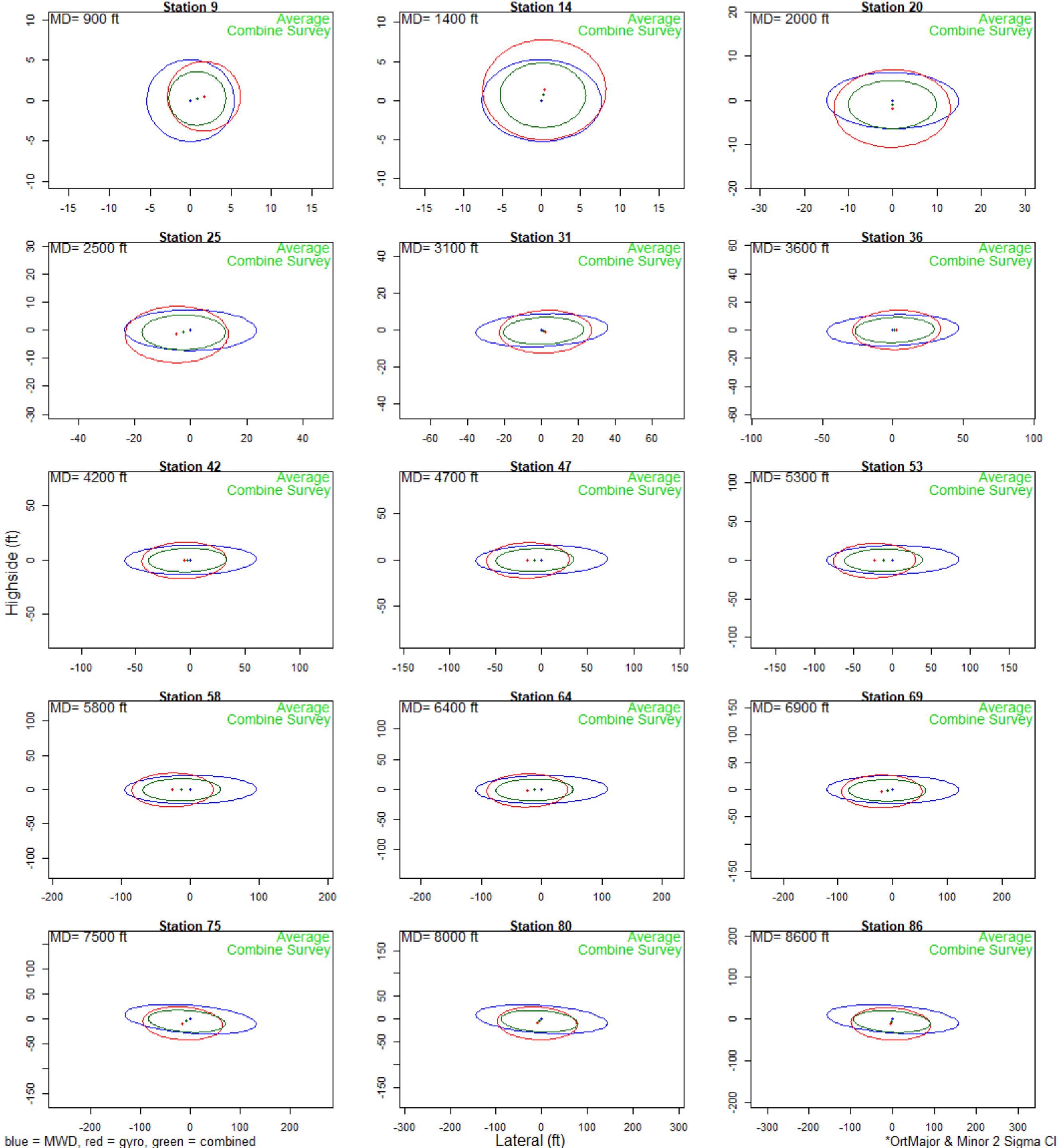
```
#################Setting up HLA plot#############
     ############for the MWD survey#############
     Hmcx<- 0 #Since MWD is the reference survey the M
     Hmcy<- 0 #Since MWD is the reference survey the M
     Hmx<- sc15p[r,8] #grabbing the ortmaj
     Hmy<- sc15p[r,9] #grabbing the ortmin
     HmC <- c(Hmcx,Hmcy) #making the ellipse center po
     Hmr<- sc15p[r,34]

     Hmell <- Ellipse$new(HmC,Hmx,Hmy,Hmr) #creating t
```



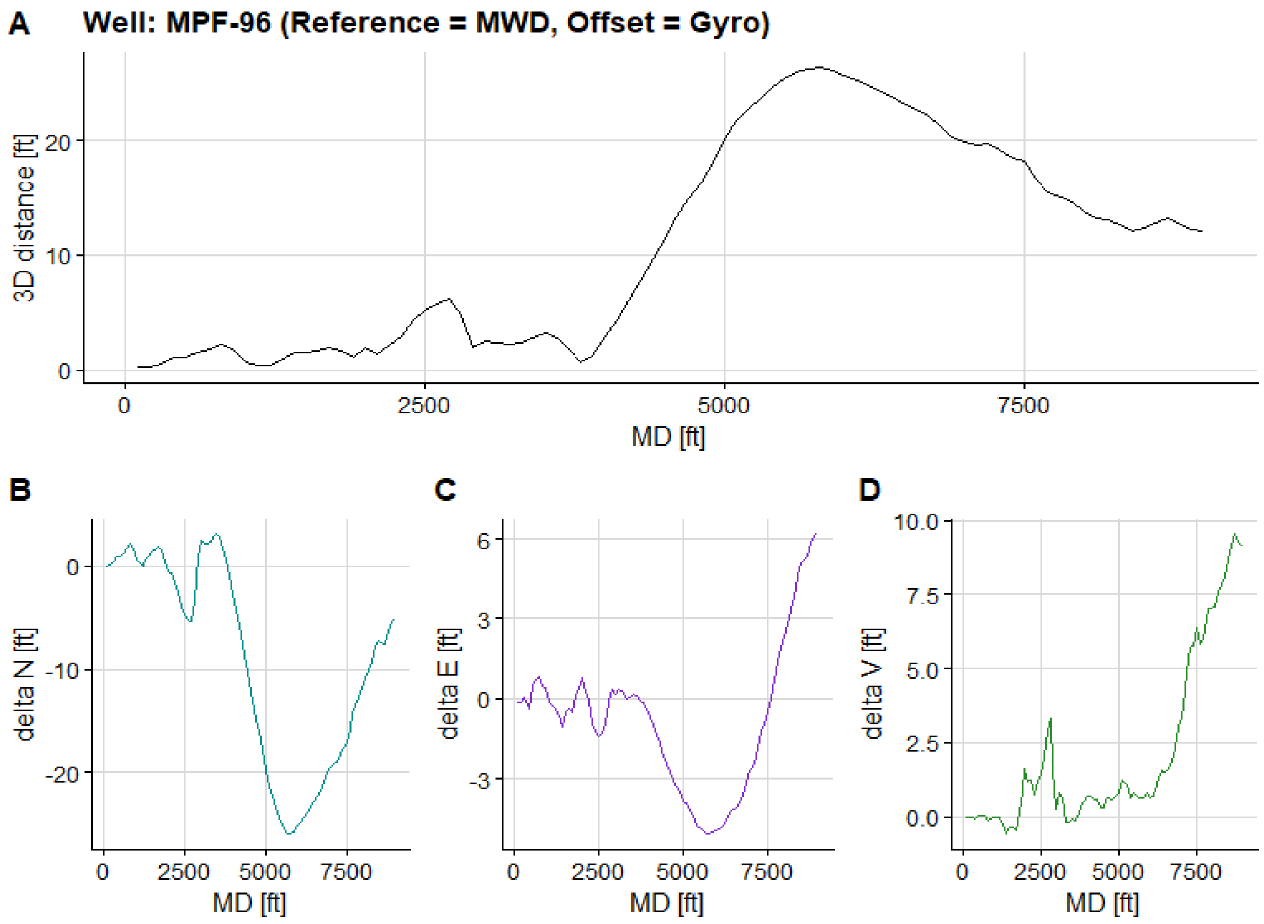

```
###############rating code####################
if (mlx < glx & glx < Hmcx & Hmcx < gux & gux < mux & mly < gly & gly < Hmcy & Hmcy < guy & guy < muy){
    vah <- "Very Good"
    #cat("Station", s,":",va, "\n")
     colorh<- "green3"
}else{
       if(mlx < glx & mlx < Hgcx & gux < mux & Hgcx< mux & mly < gly & mly < Hgcy & guy < muy & Hgcy < muy){
       vah<- "Good"
       #cat("Station", s,":",va, "\n")
       colorh<- "green3"}
  else{
    if (glx < mlx & mlx < gux | glx < mux & mux < gux | gly < mly & mly < guy | gly < muy & muy < guy){
      if(mlx < Hgcx & Hgcx < mux & mly < Hgcy & Hgcy < muy){
          vah<- "Average"
          #cat("Station", s,":",va, "\n")
           colorh<- "green3"}
        else{
          vah<- "Poor"
          #cat("Station", s,":",va, "\n")
          colorh<- "yellow2"}
  }
  else{
    vah<- "Unacceptable"
    #cat("Station", s,":",va, "\n")
    colorh<- "red"}
}
}
```

```
###############HLA plot################################
par(mar= c(4,3.8,3.5,2.1))
      plH<-{plot(NULL, asp = 1,
                 xlim = c(Hxl,Hxu), ylim = c(Hyl,Hyu), xlab = "", ylab = "") +
                 title(main =paste("Station", sn,"HLA"))+
                 title(xlab = "Lateral (ft)", line = 2)+
                 title(ylab = "Highside (ft)", line = 2)
                 mtext(paste("MD=", d,"ft"), side = 3, adj = .01, line = -1.02, cex = .9)
                 mtext(paste(vah), side = 3, adj = .99, line = -1.02, col = colorh, cex = .9)

            points(rbind(HmC), pch = 20, col = "blue",cex = 1)#MWD center
            points(rbind(HgC), pch = 20, col = "red",cex = 1)#gyro center
            points(rbind(HcC), pch = 20, col = "darkgreen",cex = 1)#combined center

            draw(Hmell, border = "blue") #draw MWD ellipse
            draw(Hgell, border = "red") #draw gyro ellipse
            draw(Hcell, border = "darkgreen") #draw combined ellipse

            mtext(paste("*OrtMajor & Minor 2 Sigma CI"), side = 3, cex = .8)
            mtext(expression(bold("B")), side = 3, cex = 1.5, adj = .55, outer = TRUE, line = -2)}
```

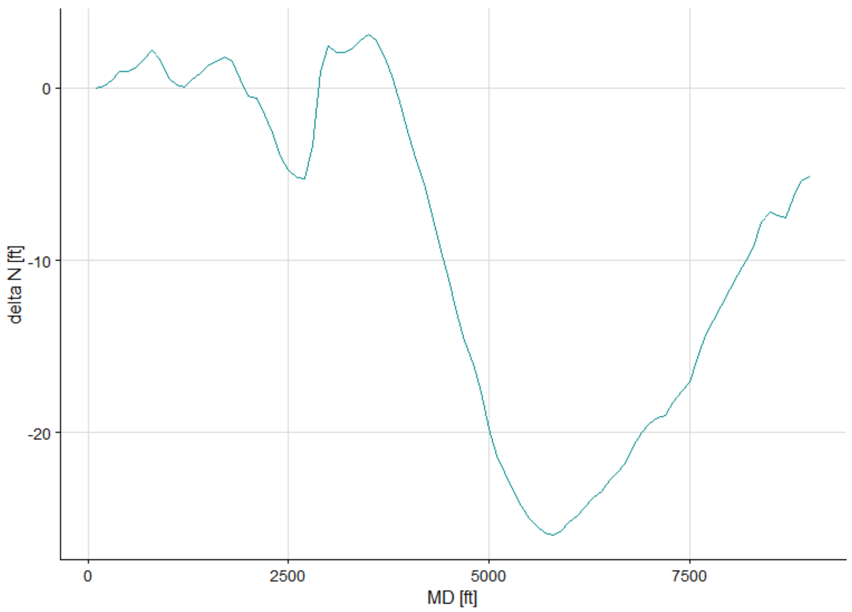
Ellipse Test

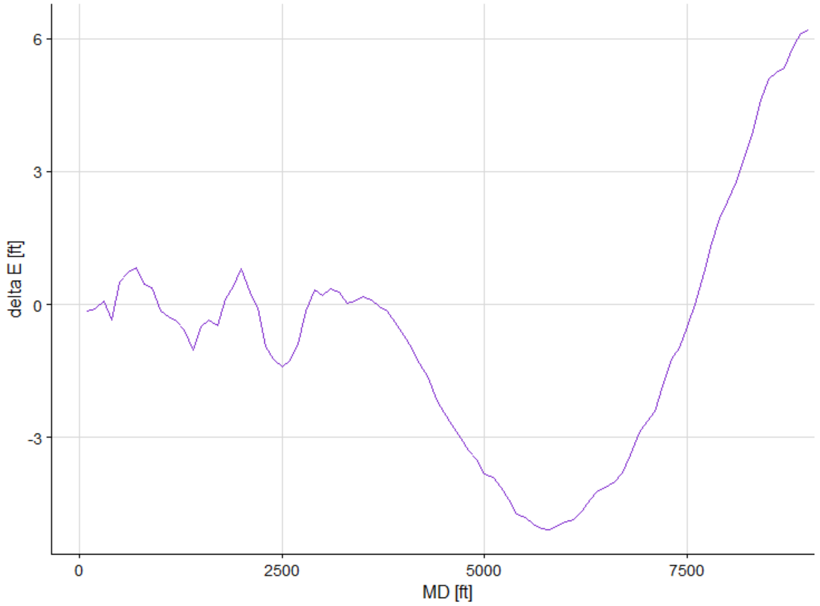


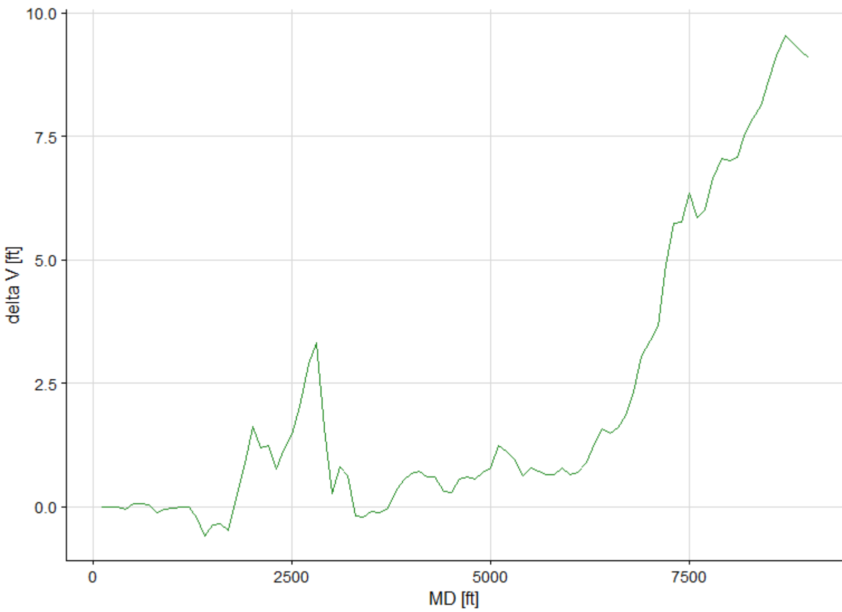
Distance

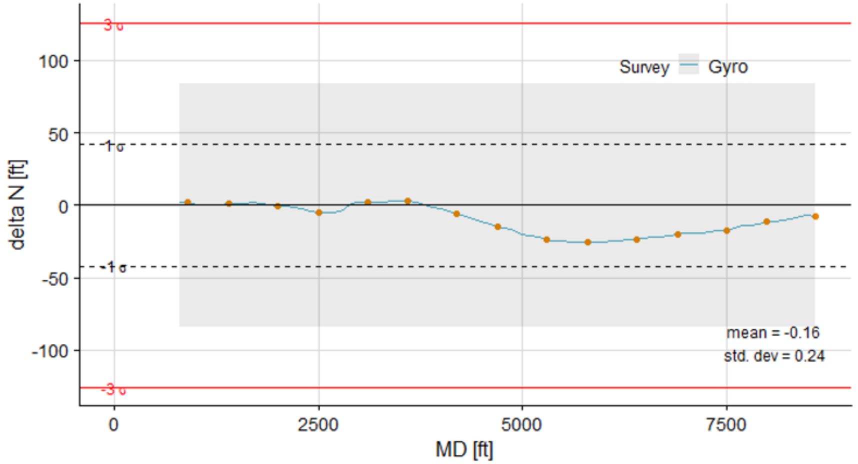

A
Well: MPF-96 (Reference = MWD, Offset = Gyro)
3D distance [ft]
0
10
20
0
2500
5000
7500
MD [ft]
B
delta N [ft]
0
-10
-20
0
2500
5000
7500
MD [ft]
C
delta E [ft]
6
3
0
-3
0
2500
5000
7500
MD [ft]
D
delta V [ft]
10.0
7.5
5.0
2.5
0.0
0
2500
5000
7500
MD [ft]

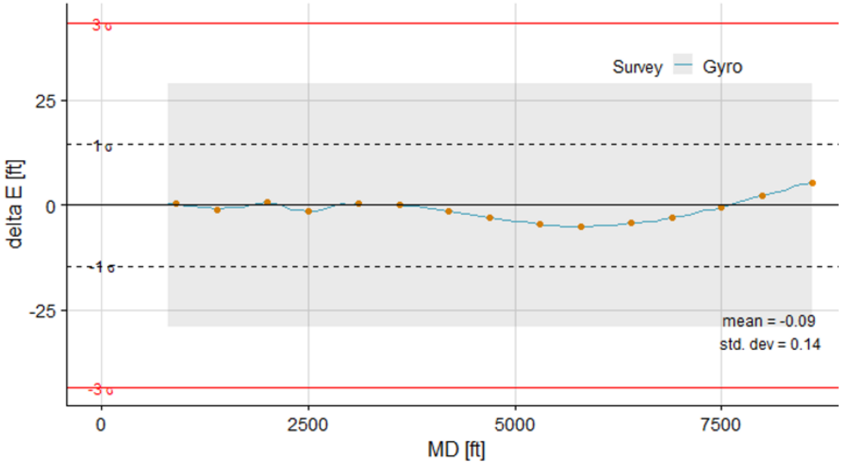
Distance RIP Plots

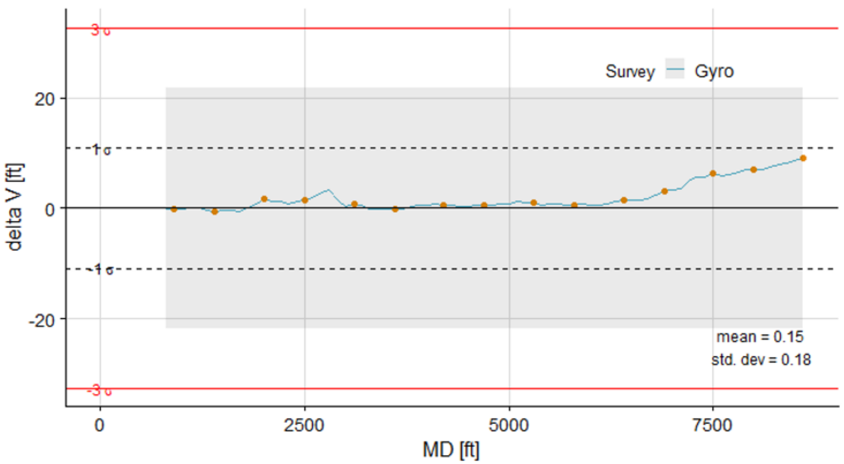












Questions?

Two Sided Chi-Square Test

Upper-tail critical values of chi-square distribution with *v* degrees of freedom

v	0.90	0.95	0.975	0.99	0.999
	Probability less than the critical value				
1	2.706	3.841	5.024	6.635	10.828
2	4.605	5.991	7.378	9.210	13.816
3	6.251	7.815	9.348	11.345	16.266
4	7.779	9.488	11.143	13.277	18.467
5	9.236	11.070	12.833	15.086	20.515
15	22.307	24.996	27.488	30.578	37.697

Lower-tail critical values of chi-square distribution with *v* degrees of freedom

v	0.10	0.05	0.025	0.01	0.001
	Probability less than the critical value				
1.	.016	.004	.001	.000	.000
2.	.211	.103	.051	.020	.002
3.	.584	.352	.216	.115	.024
4.	1.064	.711	.484	.297	.091
5.	1.610	1.145	.831	.554	.210
15.	8.547	7.261	6.262	5.229	3.483

https://www.itl.nist.gov/div898/handbook/eda/section3/eda3674.htm