Collision Risk Analysis
Pedal curve vs. Ellipse

Jon Bang
Gyrodata Ltd.

Erik Nyrnes
Statoil ASA

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Summary

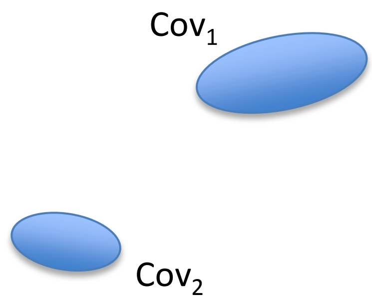
1. Select wellbore locations

X

X

Not treated here (but very important!)

2. Uncertanties



From error model

3. Combine the uncertainties

a) σ_1 and σ_2 on individual locations
b) σ on the distance between

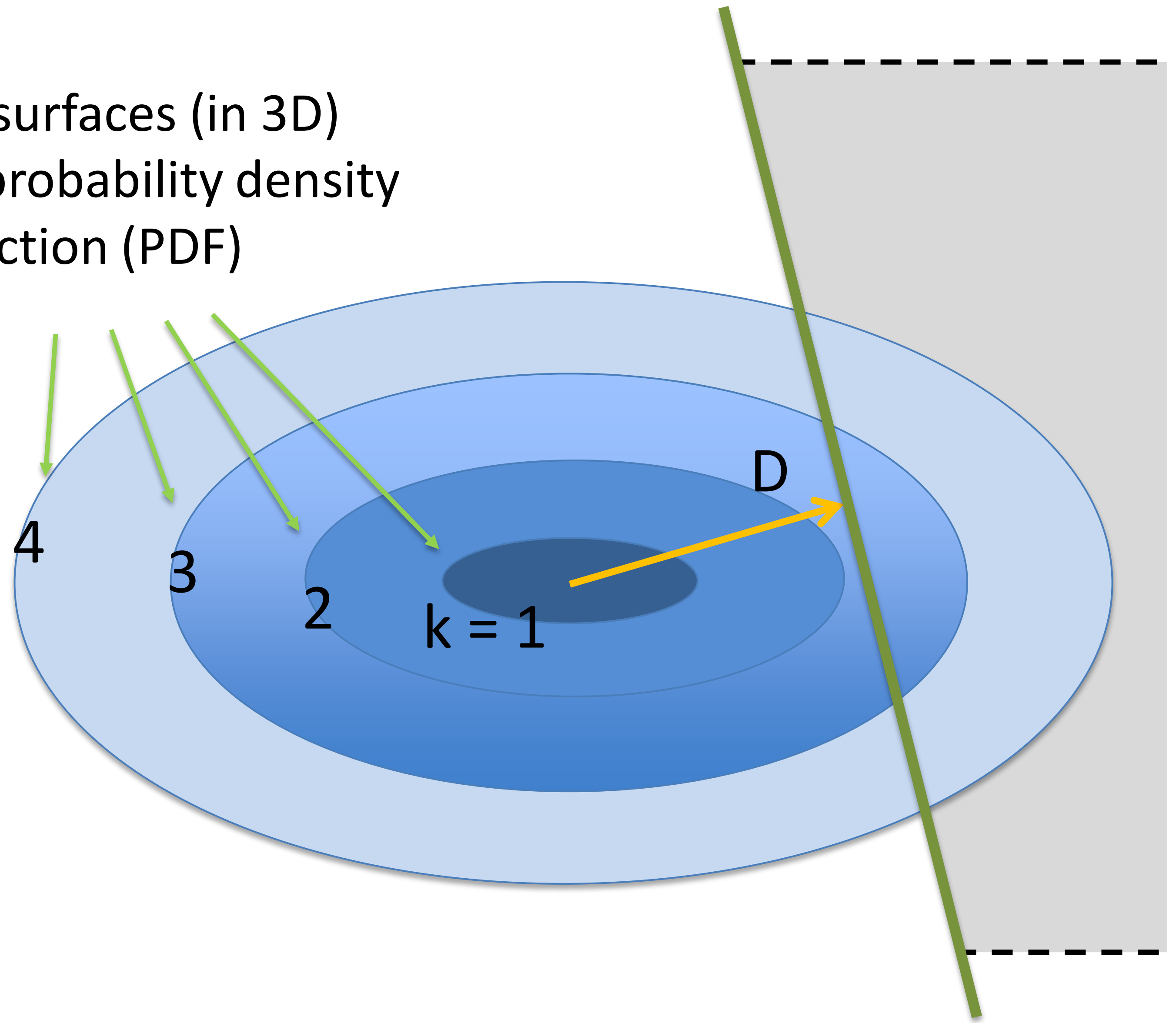
$$Cov = Cov_1 + Cov_2$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2$$

Same resulting formulas for a) and b)!

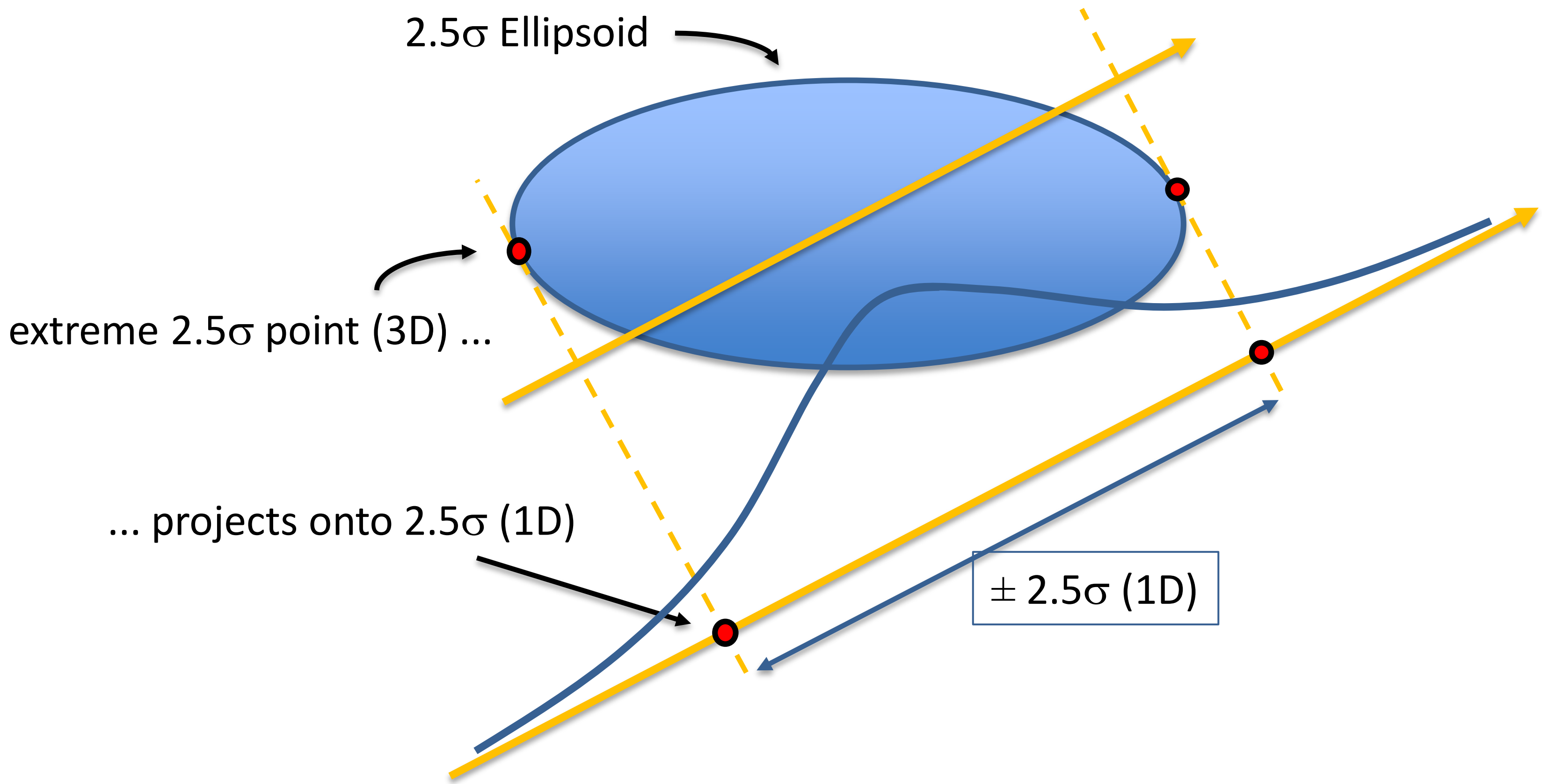
The Foundation

kσ surfaces (in 3D)
of probability density
function (PDF)



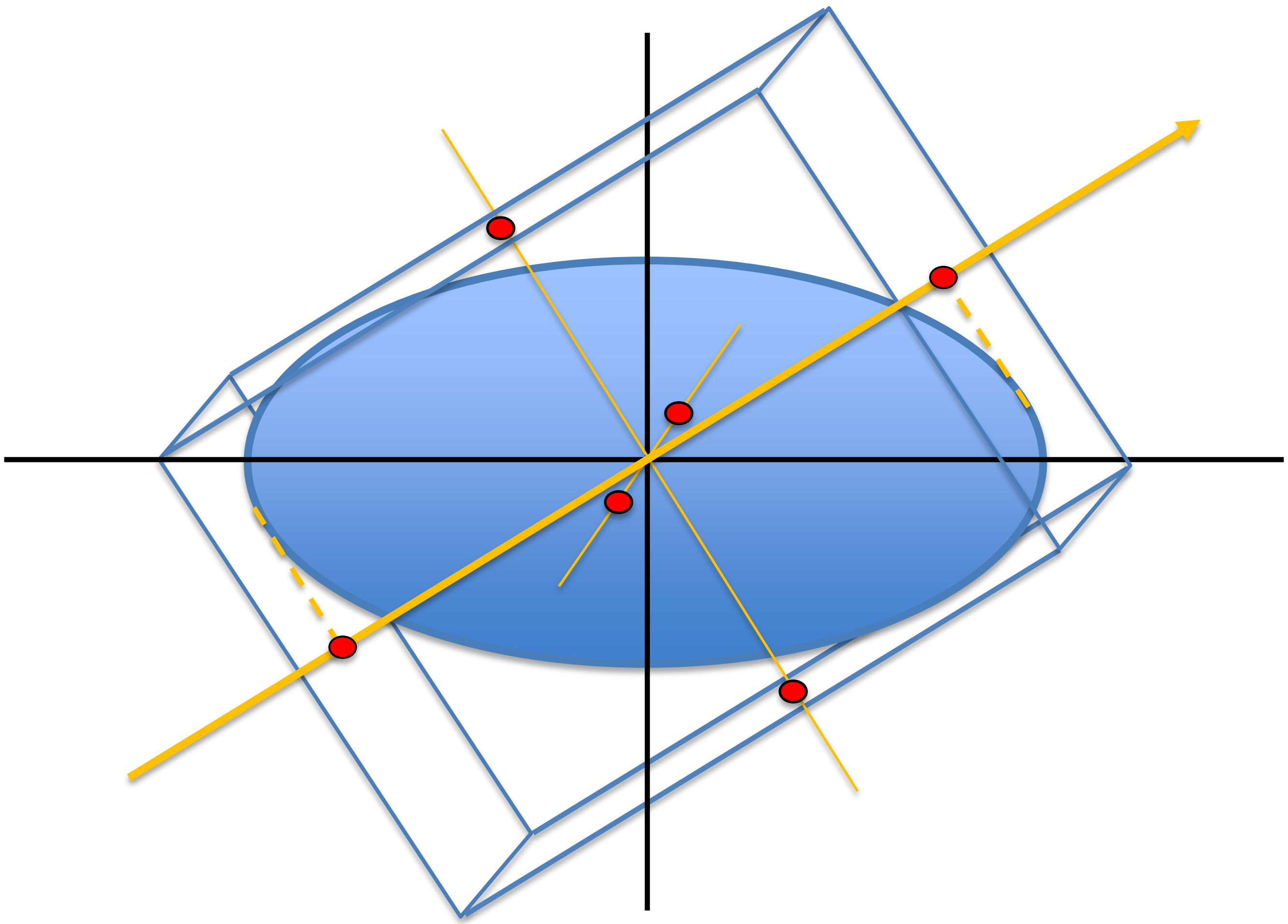
Collision / Crossing Analysis: 1D Problem

NB: example assumes k = 2.5

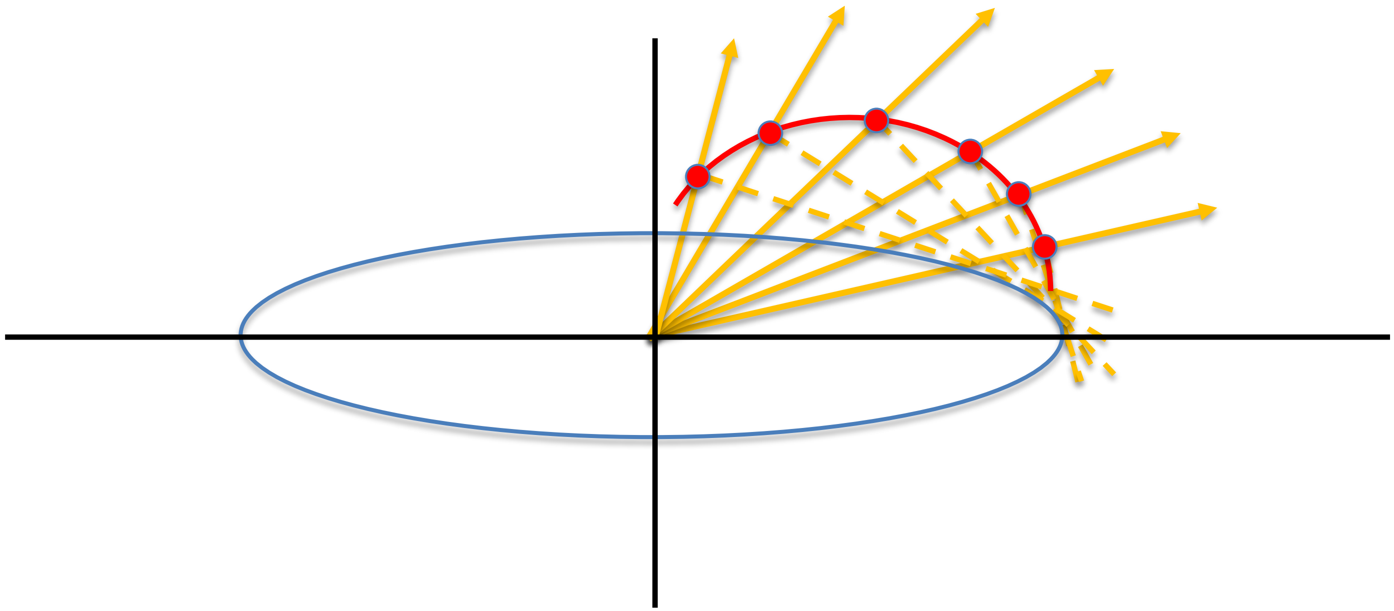


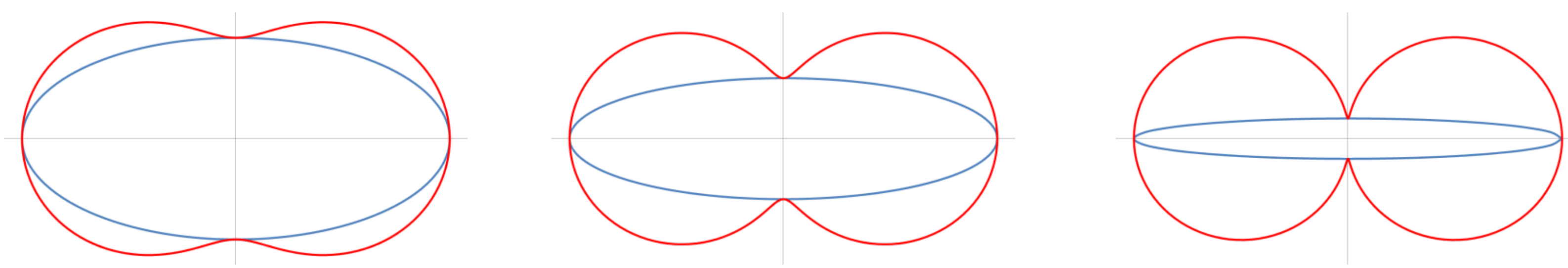
Connection between 3D, 2D, and 1D: **The same k-level**

Think «Inside the Box»



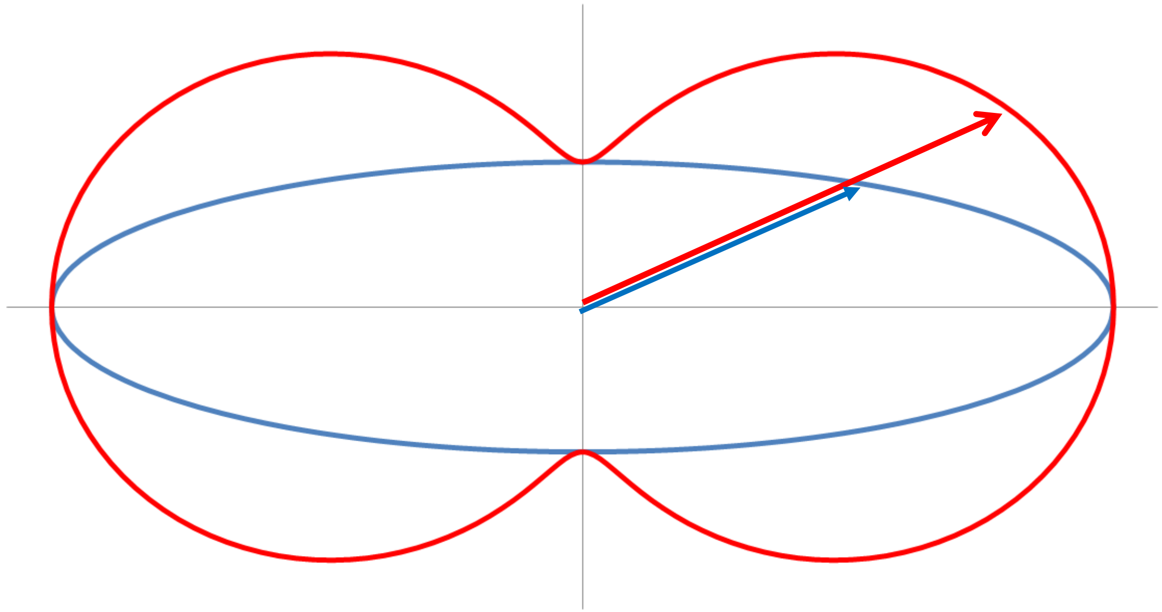
The Mysterious Pedal Curve





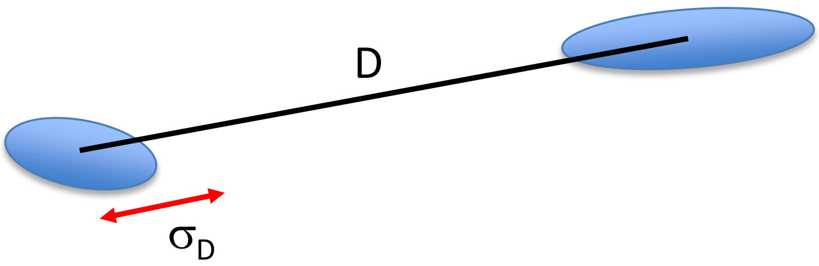
... and in 3D: Pedal Surface

The Not-So-Mysterious Pedal Curve

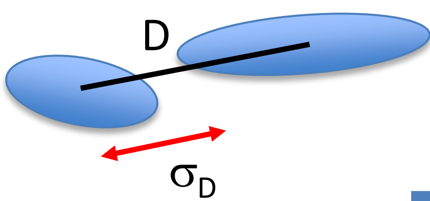


- Pedal curve radius $\geq$ Ellipse radius for all directions.
- Pedal curve radius relates to probability; ellipse radius does not.

D versus kσ_D



large k = large D = low risk



small k = small D = high risk

$$SF = \frac{D - casing\ dimensions}{k\sigma_D}$$

Company	k	Risk when SF = 1
	1.00	15.8 % (10/63)
	2.00	2.27 % (1/44)
	2.445	0.726 % (1/138)
Statoil	2.878	0.200 % (1/500)
	3.00	0.135 % (1/742)

NB: Normal PDF distribution assumed

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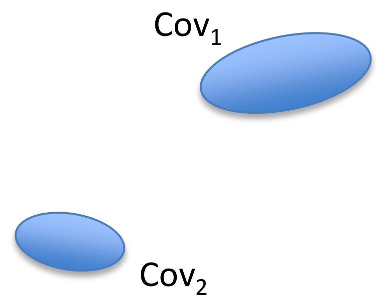

From error model

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$$Cov = Cov_1 + Cov_2$$

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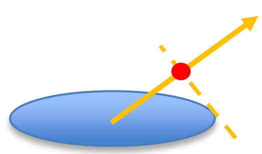
Same resulting formulas for a) and b)!

4. Interpretation of σ

Pedal point: projection from 3D (2D) onto 1D



NB: Consider only ellipse/-oid
(i.e., ignore pedal point)
=> risk cannot be quantified

5. Quantification of risk

$SF = D / k\sigma$

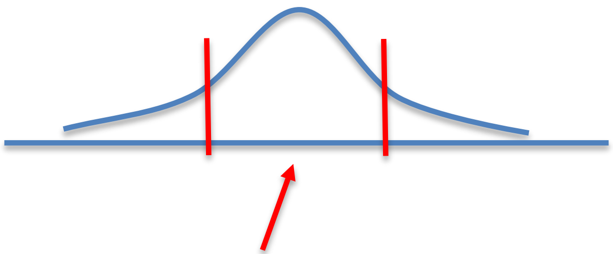
Criterion: SF = 1 (for given k & given PDF)

Normal PDF assumed here; may be another PDF (heavier tales)

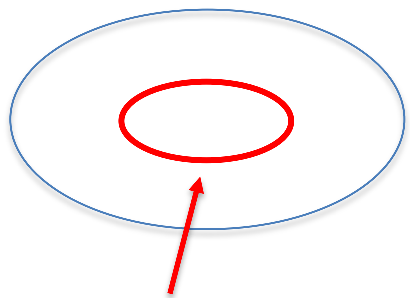
± kσ Confidence Intervals

(Normal distribution; probabilities of being **inside** an interval / curve / surface)

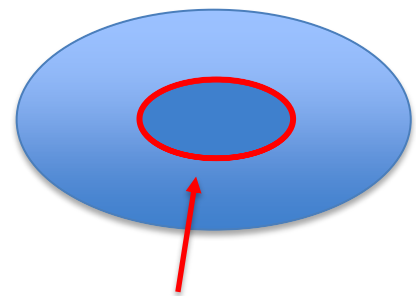
1D



2D

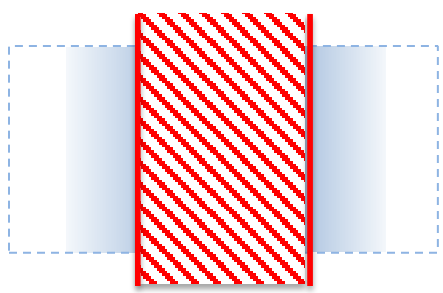


3D

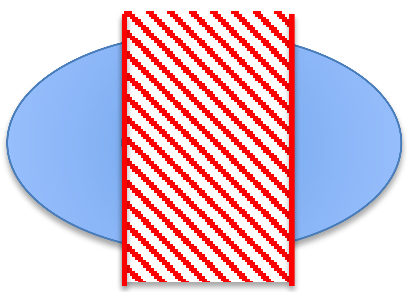


1D	2D	3D
± 1.00 σ = 68.27 %	± 1.00 σ = 39.35 %	± 1.00 σ = 19.87 %
± 2.00 σ = 95.45 %	± 2.00 σ = 86.47 %	± 2.00 σ = 73.85 %
± 2.58 σ = 99.01 %	± 2.58 σ = 96.41 %	± 2.58 σ = 91.63 %
± 2.79 σ = 99.47 %	± 2.79 σ = 97.96 %	± 2.79 σ = 94.93 %
± 3.00 σ = 99.73 %	± 3.00 σ = 98.89 %	± 3.00 σ = 97.07 %

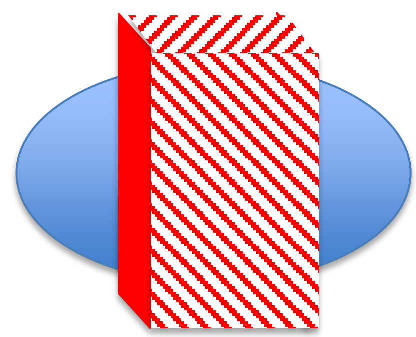
Why the differences?



68.27 %



68.27 %



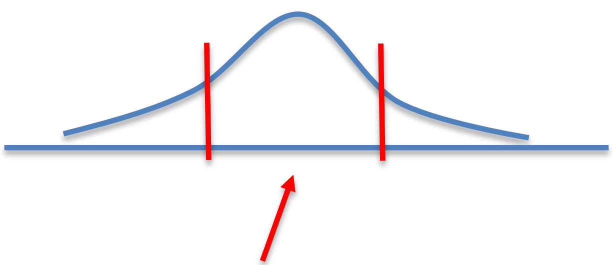
68.27 %

± kσ Confidence Intervals

(Normal distribution; probabilities of being **inside** an interval / curve / surface)

1D	2D	3D

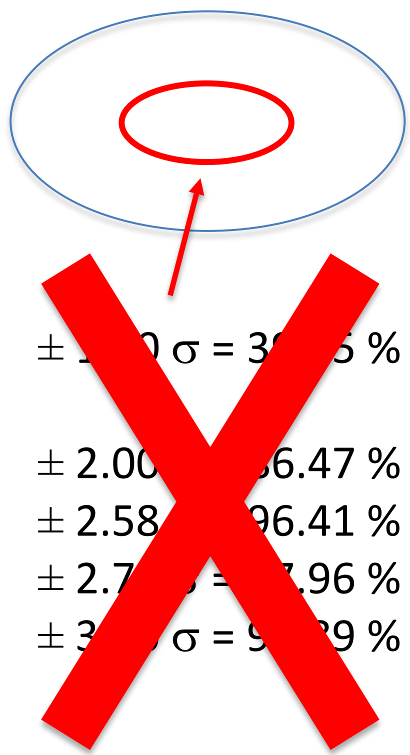
1D



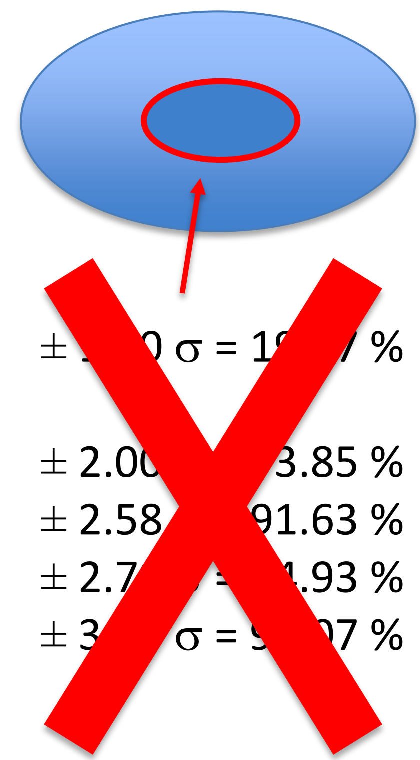
± 1.00 σ = 68.27 %

± 2.00 σ = 95.45 %
± 2.58 σ = 99.01 %
± 2.79 σ = 99.47 %
± 3.00 σ = 99.73 %

2D



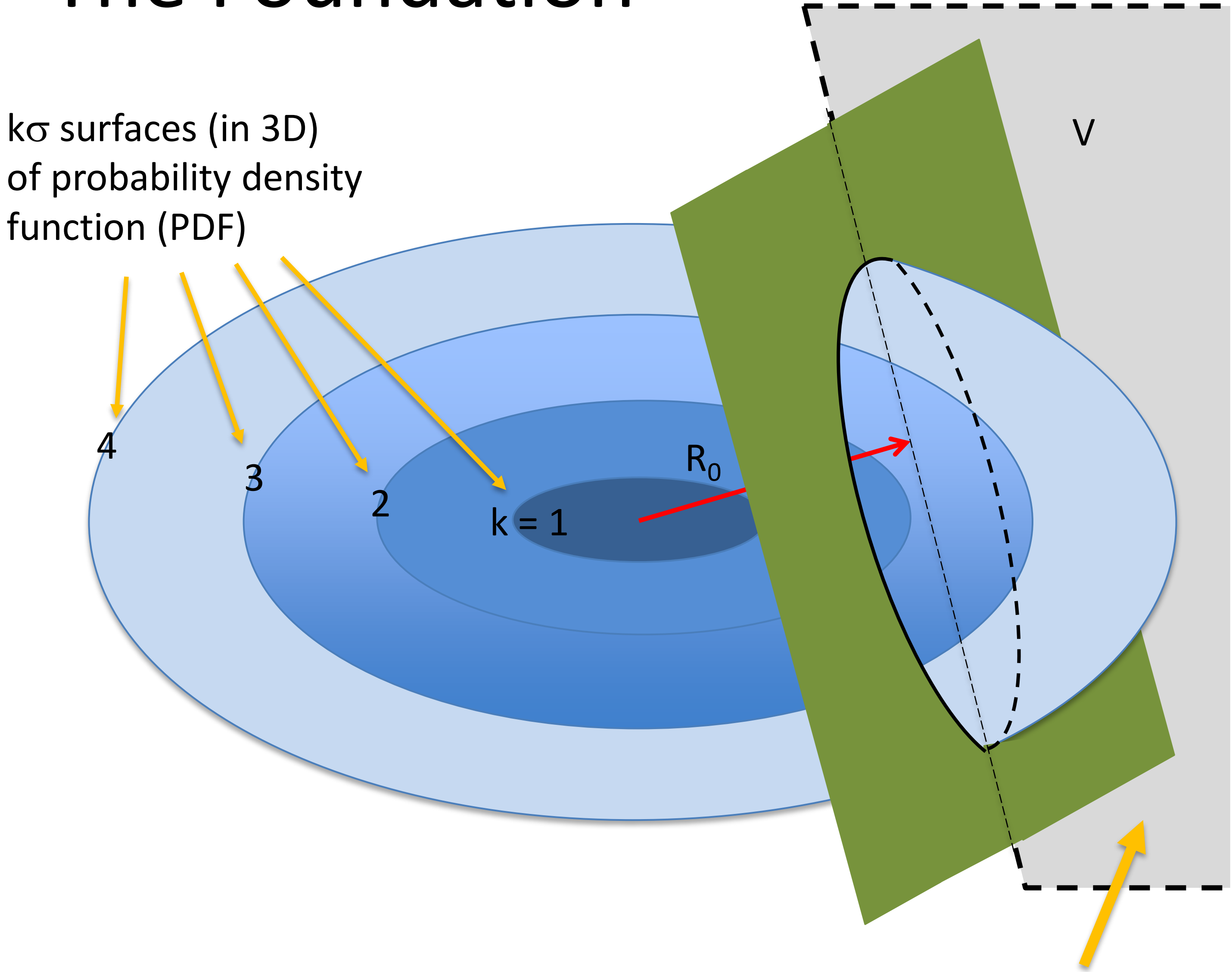
3D



Probability of collision/crossing is a 1D problem

=> never mind 2D and 3D confidence levels

The Foundation



Probability of being in a certain region V
= integral of PDF*dV over V

Two Points With Uncertainty. There are Two Ellipsoids ...?

1a. Consider either the uncertainties in both locations ....

1b. ... or the uncertainty on the distance between the locations (co-ordinate differences). This is the «Combined Cov method» in Compass.

2. The «total» Cov and σ (along connection line) become the same for 1a and 1b!

$$\left.\begin{aligned} Cov &= Cov_1 + Cov_2 \\ \sigma^2 &= \sigma_1^2 + \sigma_2^2 \end{aligned}\right\} (*)$$

3. The resulting Cov matrix gives a new ellipsoid (possibly tilted from the two original ellipsoids) that comprises the total uncertainty.

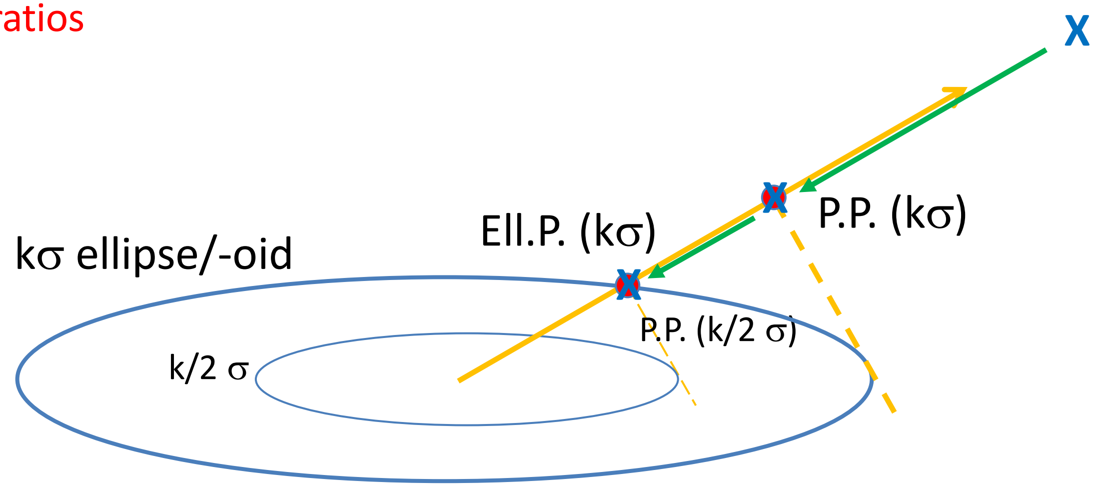
4. Locate this new ellipsoid at one well position.

5. Go on with Pedal Point, etc.

(*) assuming independency; if not, small corrections apply

If Risk Matters …

NB: example designed
with nice 2:1 ratios



At kσ-level, the Pedal Point is half way to the other well X (and the ellipse point ¼).
How close can the wells be, if we accept a risk of collision/crossing at kσ-level?

<u>Bring X to P.P.(kσ)</u>
=> risk(coll./cross) = probability of being outside kσ-level. **=> OK, just what we accept**

<u>Bring X to ellipse point (kσ),</u> which is the same point as P.P. (k/2 σ)
=> risk(coll./cross) = probability of being outside k/2 σ-level. **=> RISK HIGHER THAN kσ**

Statoil's Anti-Collision Criterion

2.1.5 Anti-collision criteria

Anti-collision calculations shall be calculated according to a Statoil approved methodology and risk level, and only Statoil approved software shall be used for such calculations.

The minimum allowed as-surveyed separation (3D borehole centre-to-centre distance) between two wells according to the requirements shall be calculated as a function of the position uncertainties and the dimensions of the actual wellbores.
The risk-based well separation rule is represented by the Separation Factor (SF) and is defined:

$$SF = \frac{D - \frac{d_1 + d_2}{2}}{2.878 \times \sigma_D}$$

Where

D = centre-to-centre distance (3D) between the reference and offset wells
σ_D = standard deviation of D
d_1, d_2 = wellbore diameters (casing or open-hole diameter at point of interest)

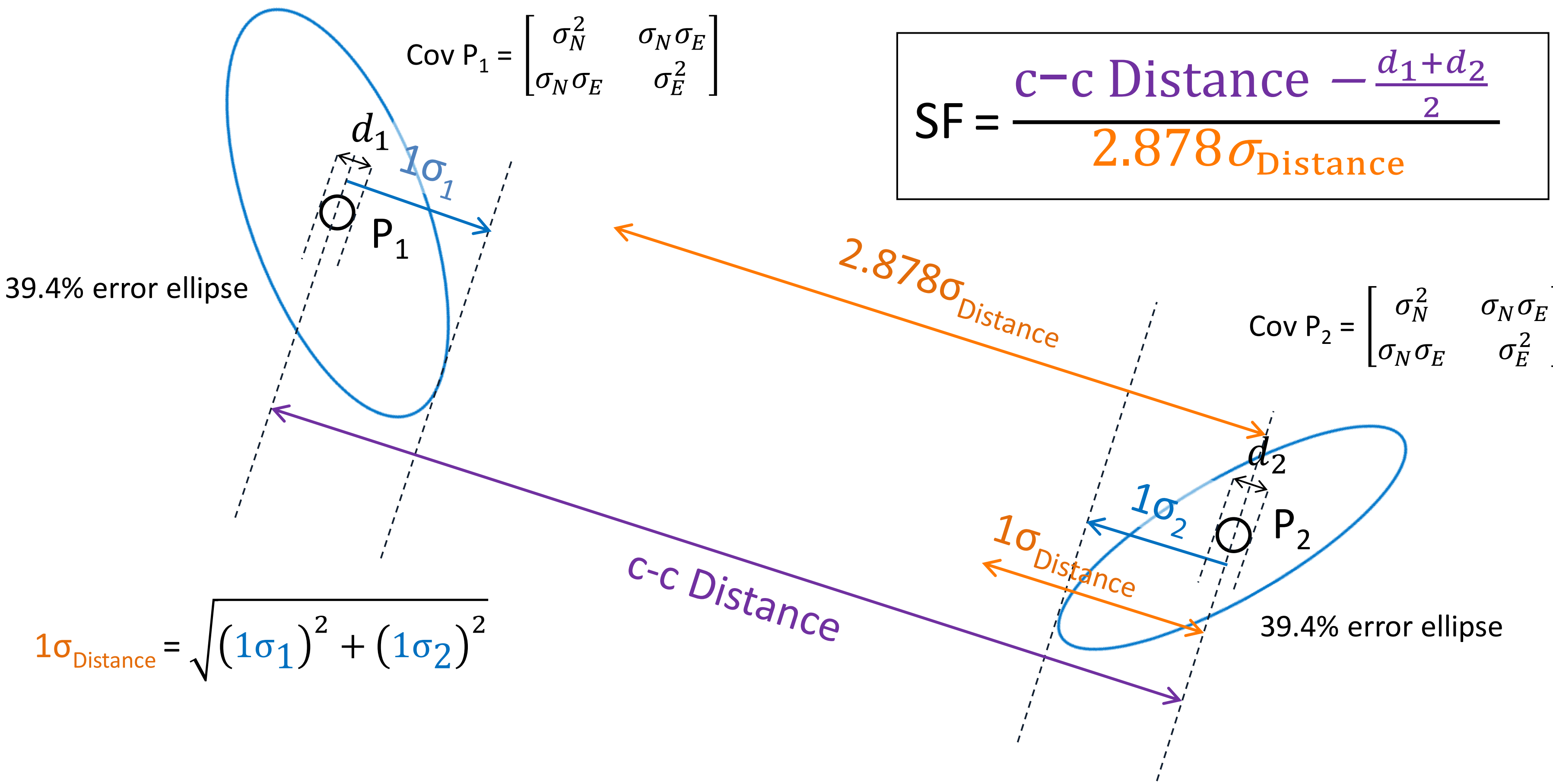
The well separation requirement is:

SF > 1

The well separation rule shall apply to all situations where an interfering well constitutes a significant risk of serious injury, fatalities, surface pollution or serious reputation damage from any other cause.

Figurative Definition of SF by the Pedal Curve Method

$$SF = \frac{\text{c−c Distance} - \frac{d_1+d_2}{2}}{2.878\,\sigma_{\text{Distance}}}$$



$$\text{Cov } P_1 = \begin{bmatrix} \sigma_N^2 & \sigma_N\sigma_E \\ \sigma_N\sigma_E & \sigma_E^2 \end{bmatrix} \qquad \text{Cov } P_2 = \begin{bmatrix} \sigma_N^2 & \sigma_N\sigma_E \\ \sigma_N\sigma_E & \sigma_E^2 \end{bmatrix}$$

$$1\sigma_{\text{Distance}} = \sqrt{(1\sigma_1)^2 + (1\sigma_2)^2}$$

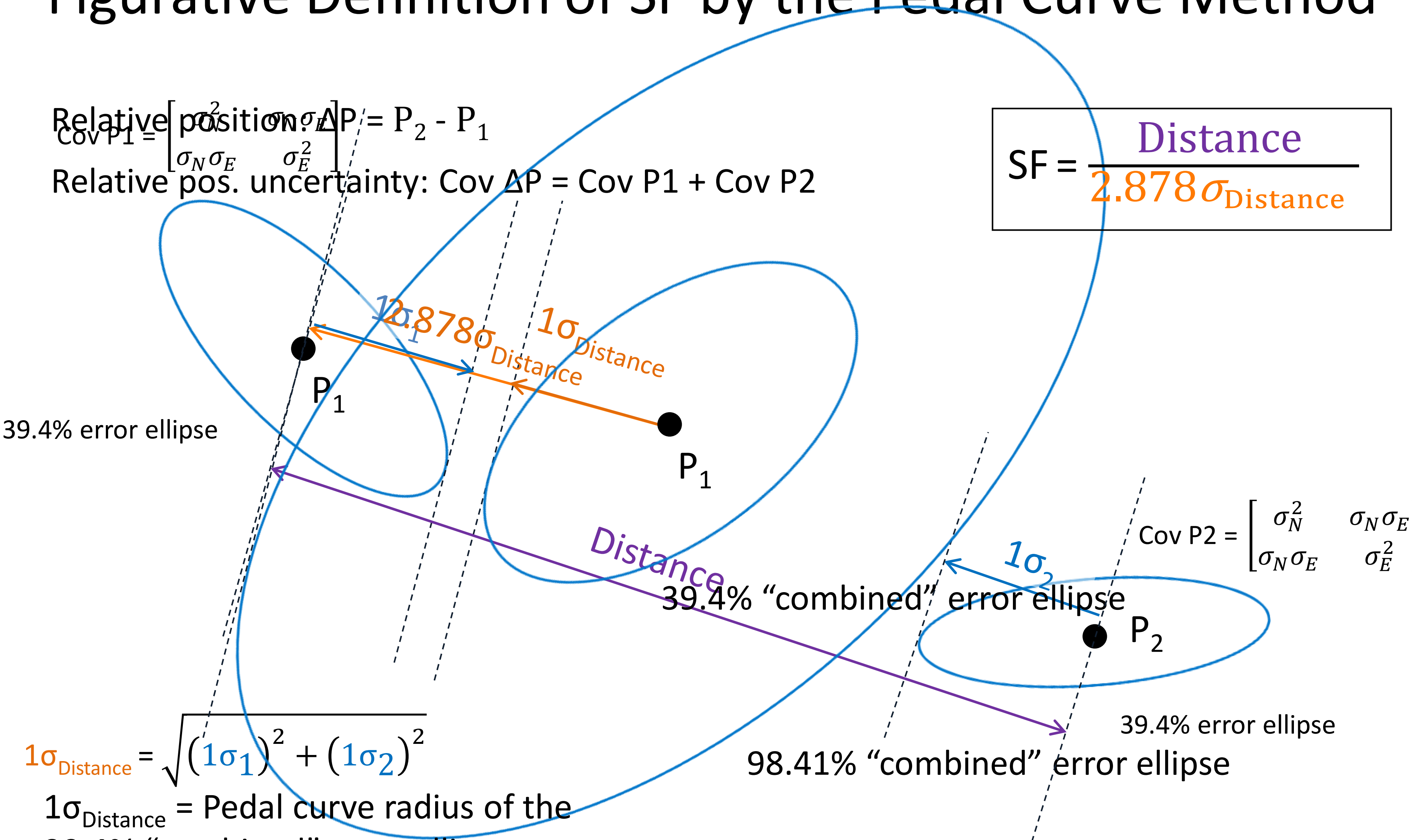
The scaling factor of 2.878 gives a probability of collision or an unintended well crossing of 1/500 when SF = 1

Figurative Definition of SF by the Pedal Curve Method

Relative position: $\Delta P = P_2 - P_1$

Relative pos. uncertainty: Cov ΔP = Cov P1 + Cov P2

$$SF = \frac{\text{Distance}}{2.878\sigma_{\text{Distance}}}$$



$$1\sigma_{\text{Distance}} = \sqrt{(1\sigma_1)^2 + (1\sigma_2)^2}$$

$1\sigma_{\text{Distance}}$ = Pedal curve radius of the 39.4% "combined" error ellipse

Information Needed to Calculate SF by the Pedal Curve Method

$$\text{SF} = \frac{\text{Distance} - \frac{d_1+d_2}{2}}{k \cdot \sigma_{\text{Distance}}}$$

- Two candidate points P_1 and P_2 in the reference well and the offset well
- Covariance matrices Σ_{p_1} and Σ_{p_2} of the two points
- Diameters d_1 and d_2 of the reference well and the offset well
- Scaling factor *k* to provide desired risk level
- Equation to calculate σ_D

NB! Be sure that the correct equation for σ_D is used:

$$\sigma_D^2 = \frac{1}{D^2}(p_1 - p_2)^T(\Sigma_{p_1} + \Sigma_{p_2})(p_1 - p_2)$$