Explicit Calculation of Expansion Factors for Collision Avoidance between Two Co-planar Survey Error Ellipses

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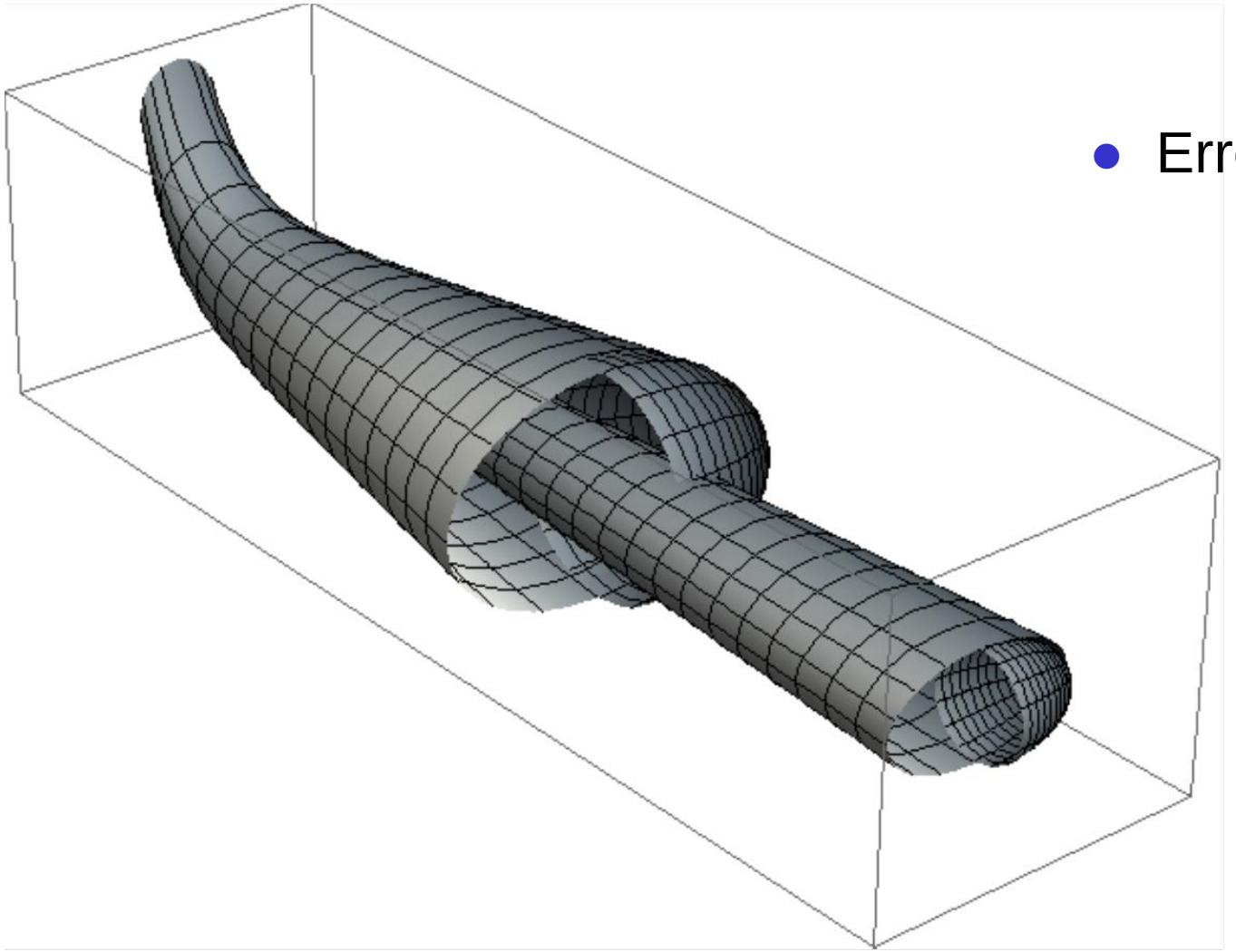
Introduction

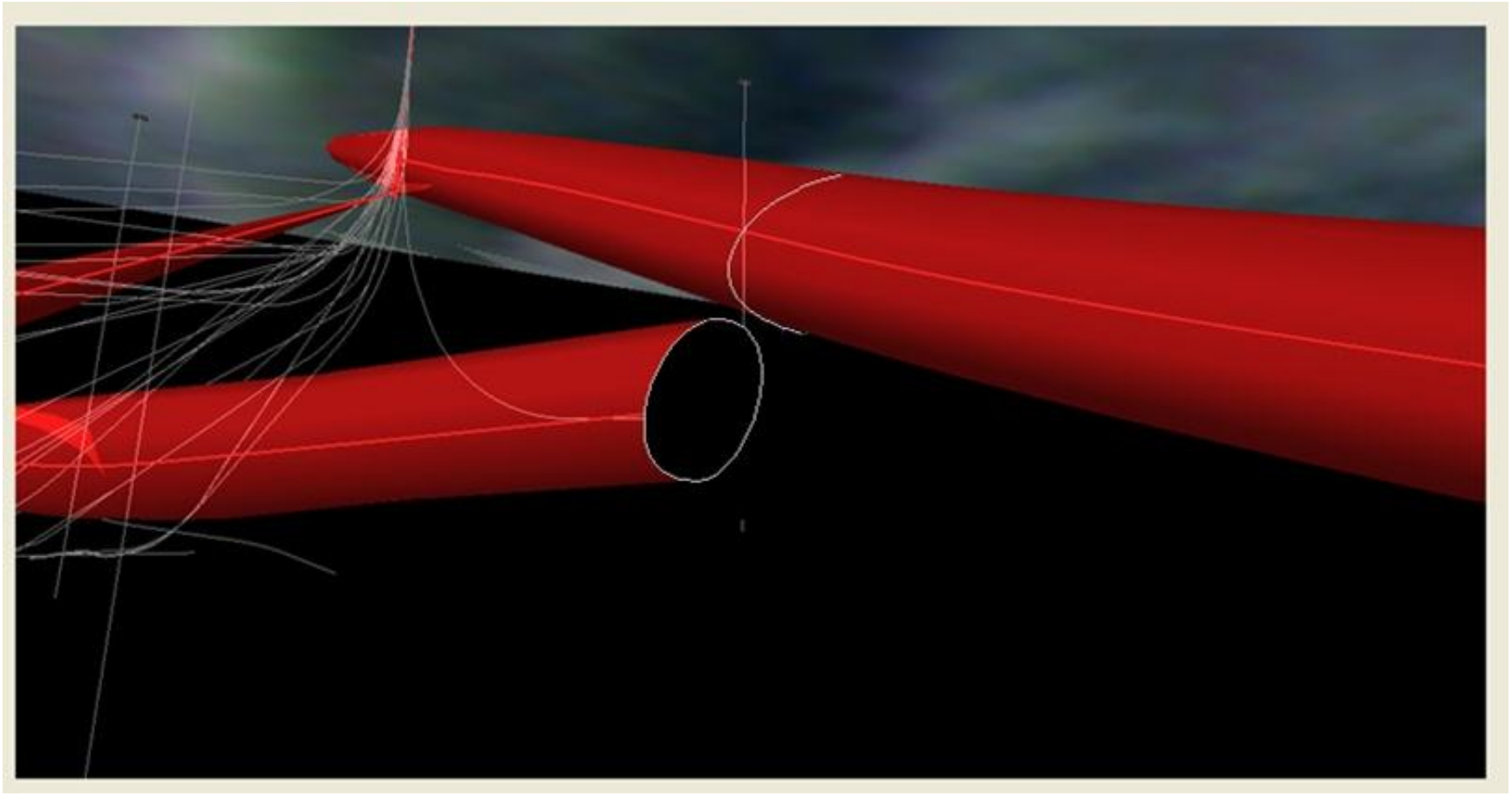
- **Separation factors currently used to represent well proximity**
  - Based on ellipses
  - Are an approximation
  - Computationally efficient
  - Easy to understand and interpret
- **Propose a like-for-like replacement (Expansion Factor)**
  - Based on ellipses
  - Are geometrically exact
  - Maintain (or enhance) computational efficiency
- **Provide a toolkit**

Positional Uncertainty: Ellipsoids



- Error models

2D Representation of 3D Separations



NORSOK D-010 Standard

Defines the model and acceptance criteria for the separation between two wellbores.

$$SF = \frac{\delta}{E_r + E_o + R_r + R_o}$$

Where:

SF = separation factor

δ = distance between the centres of the two wells

E_r = ellipse *radius* of ref. well

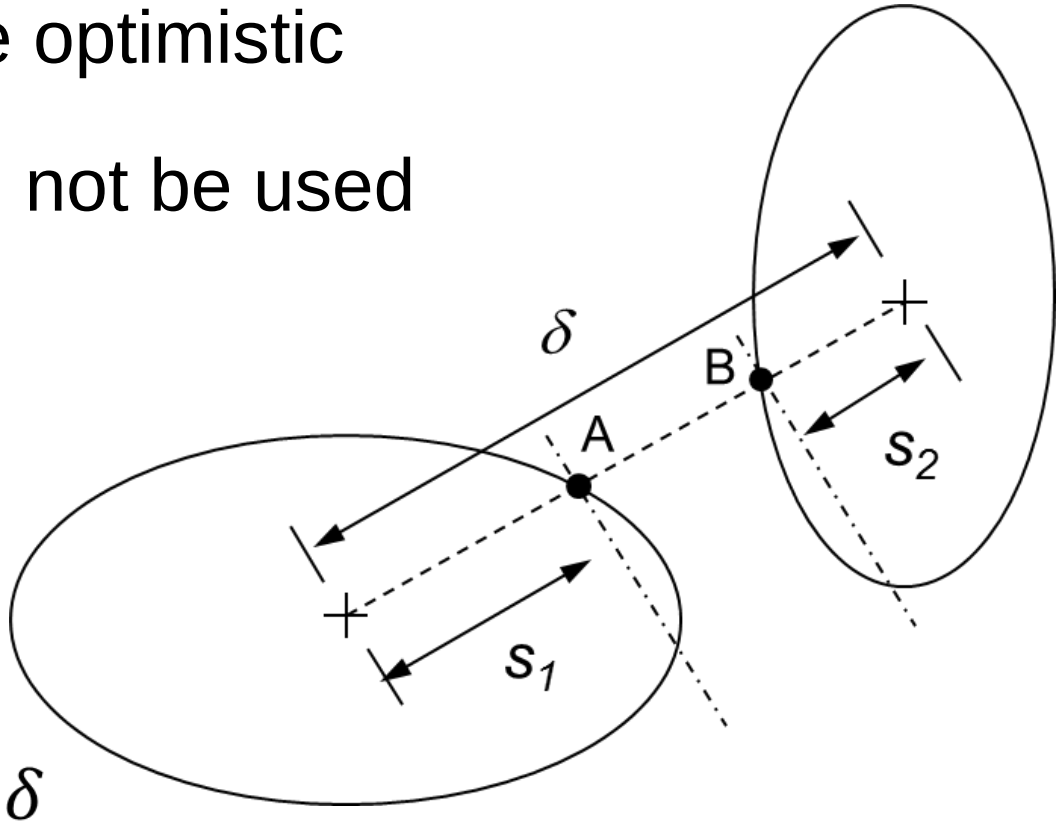
E_o = ellipse *radius* of object well

R_r = bit radius of ref. well

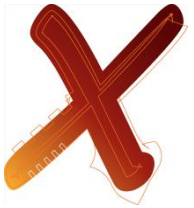
R_o = bit radius of object well

Centre Vector Method (CVM)

- Can be optimistic
- Should not be used

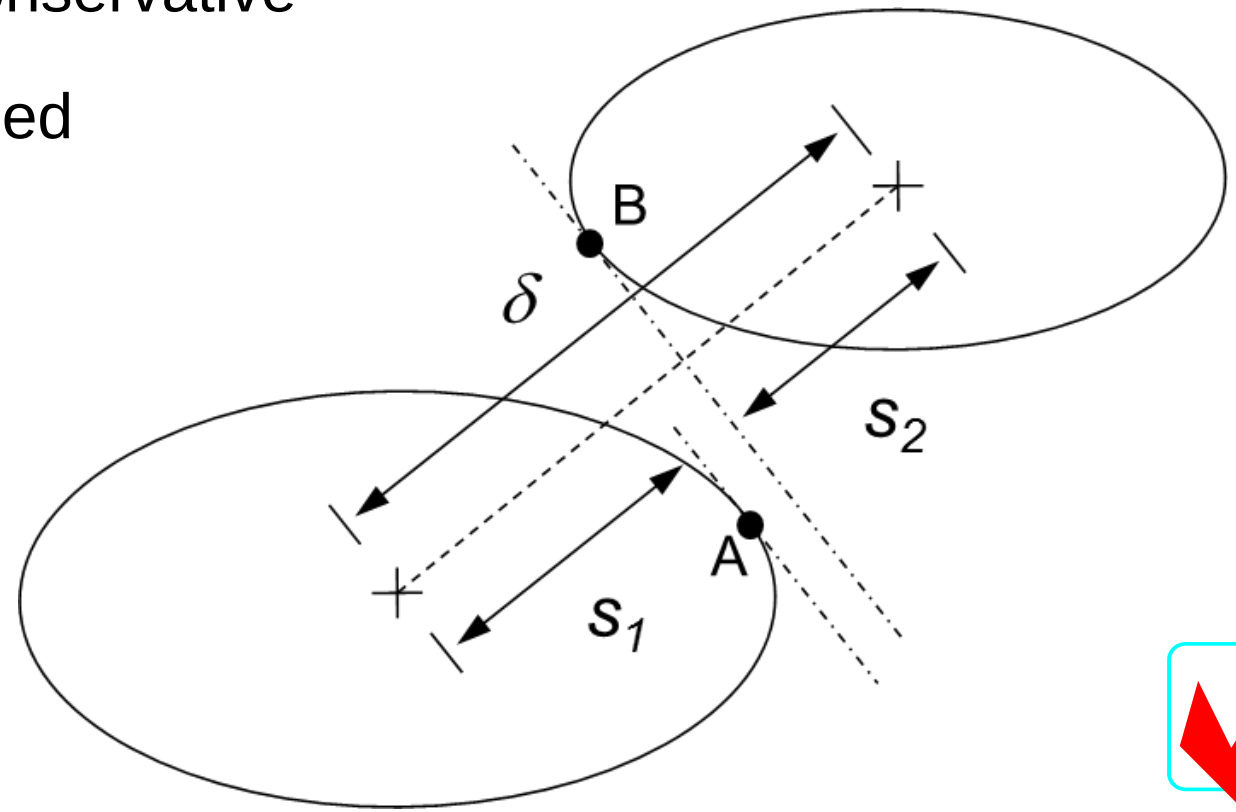


$$k_{CVM} = \frac{\delta}{S_1 + S_2}$$



Pedal Curve Method (PCM)

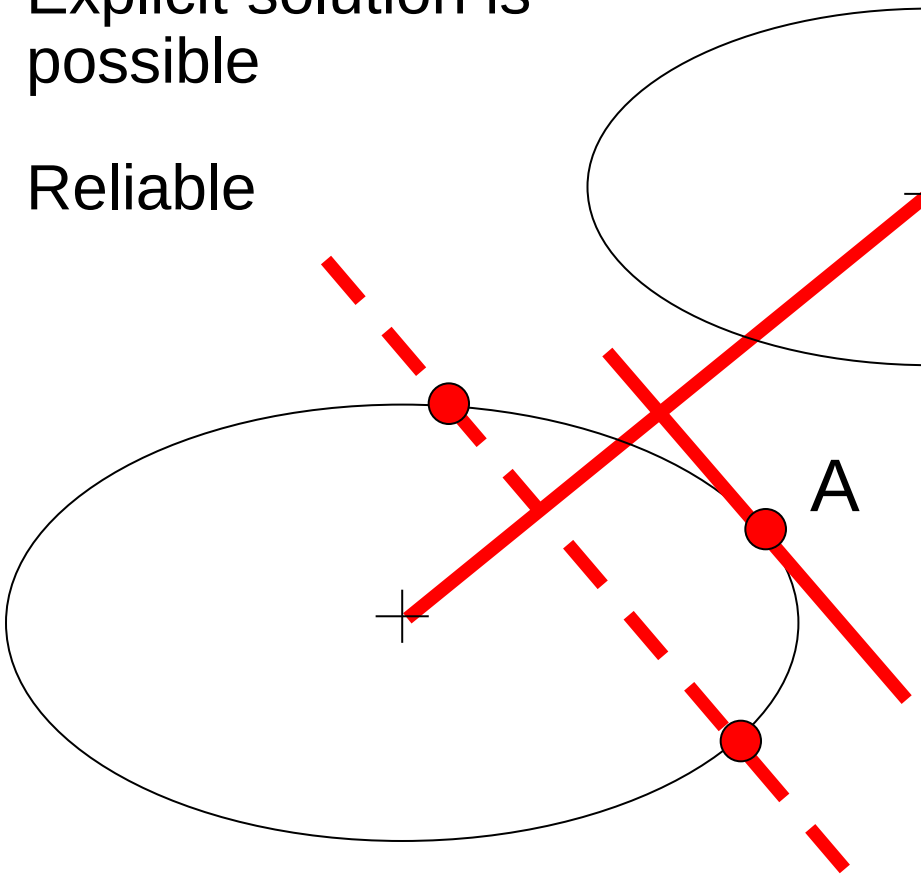
- May be conservative
- Can be used



$$k_{PCM} = \frac{\delta}{S_1 + S_2}$$

Quadratic Discriminant = 0

- Explicit solution is possible
- Reliable

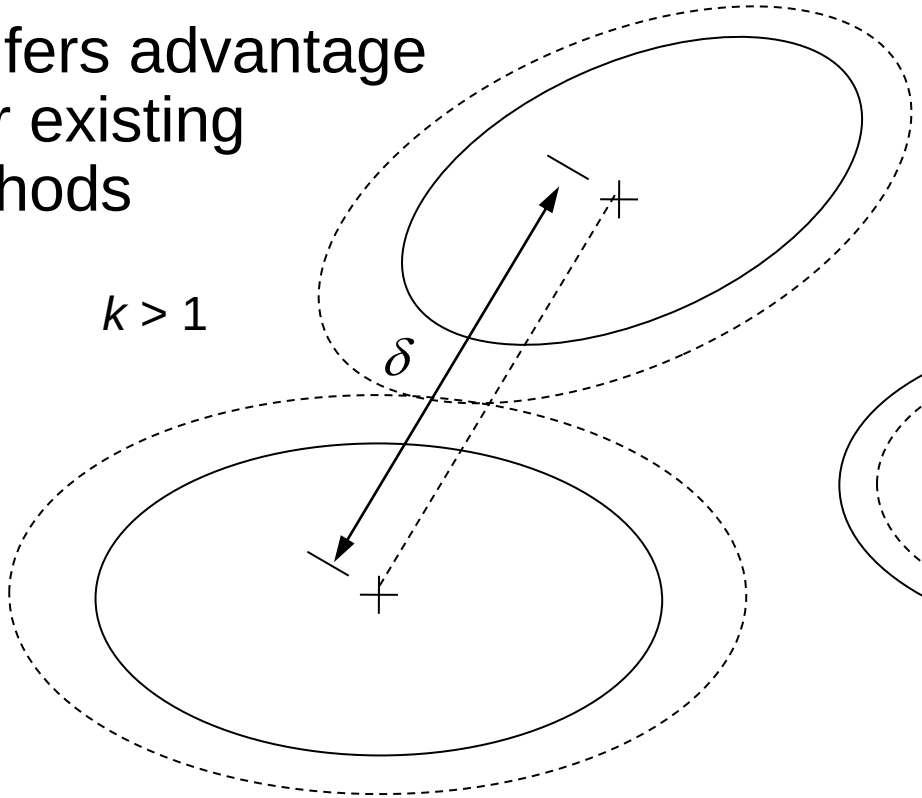


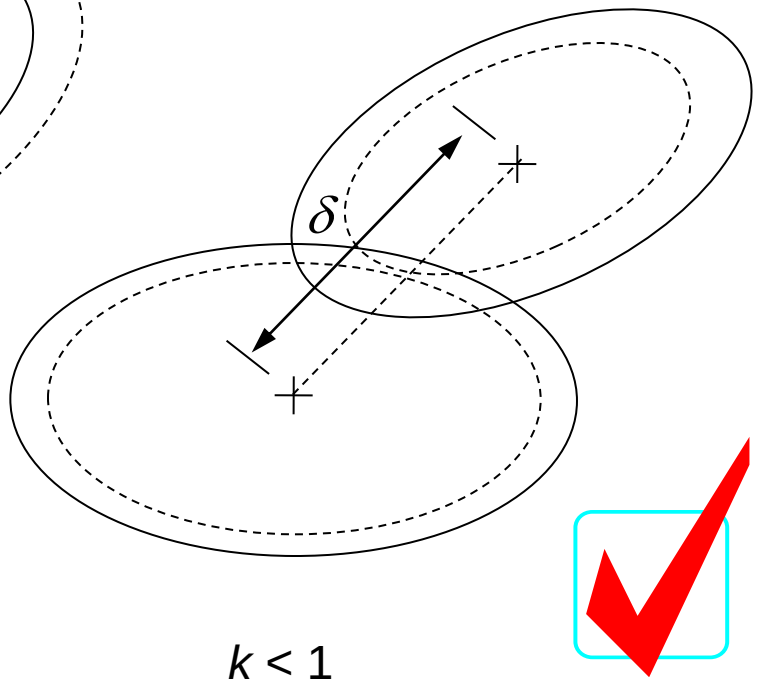
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b}{2a}$$

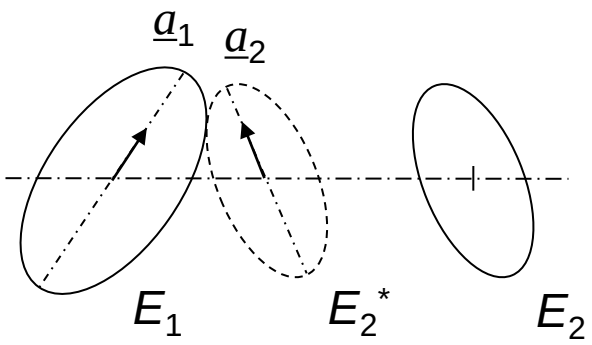
Two-Sided Expansion Factor

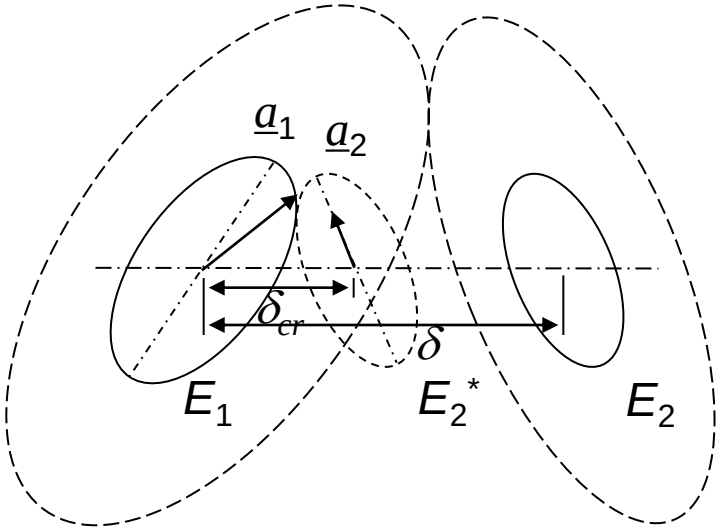
- Neither optimistic nor conservative
- Confers advantage over existing methods





Zheng & Palffy-Muhoray (ZPM)

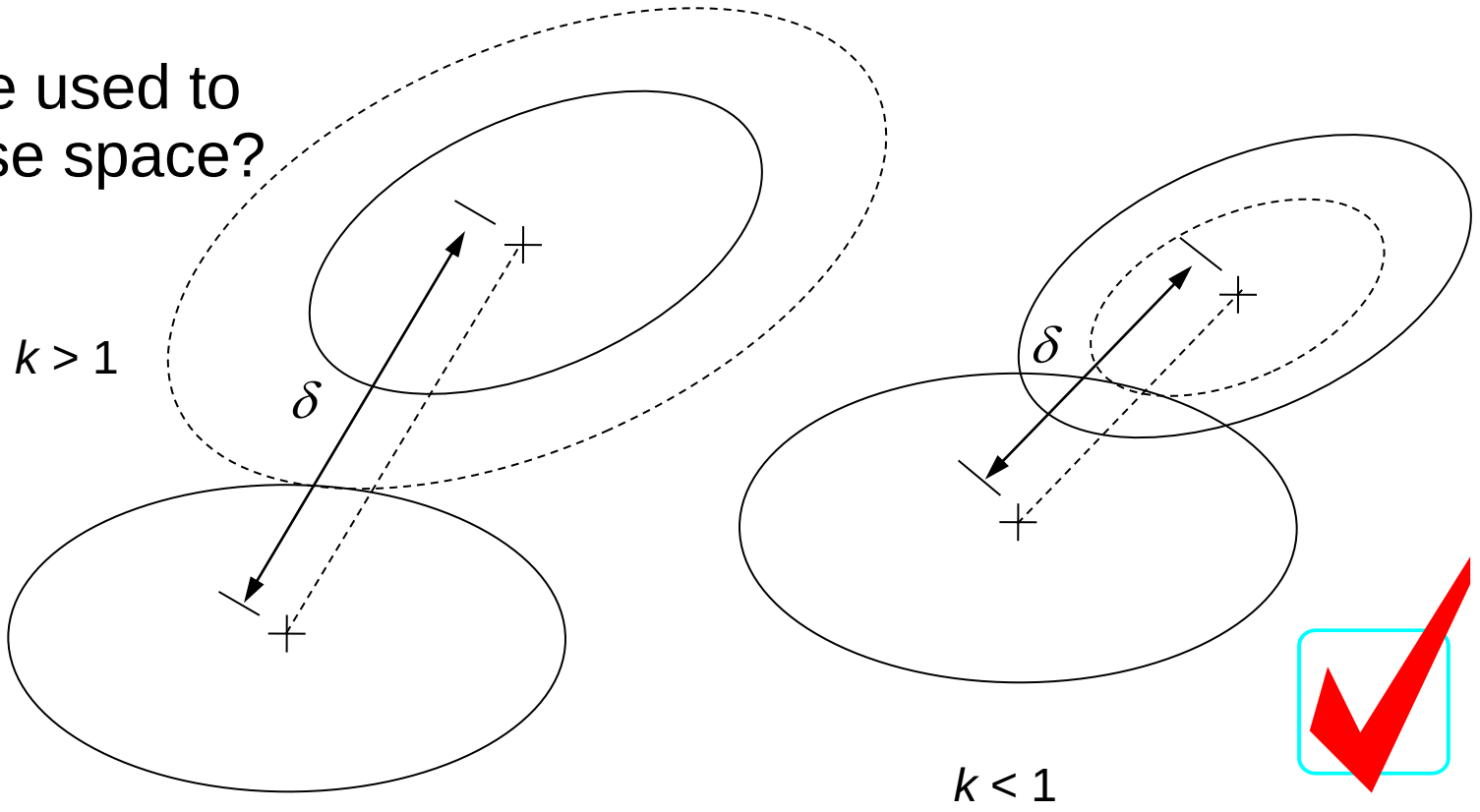




- Crystallographic studies
- ZPM give the distance of closest approach
- Two-sided expansion is equivalent to an affine transform
- The expansion factor k is proportional to the computed scaling factor

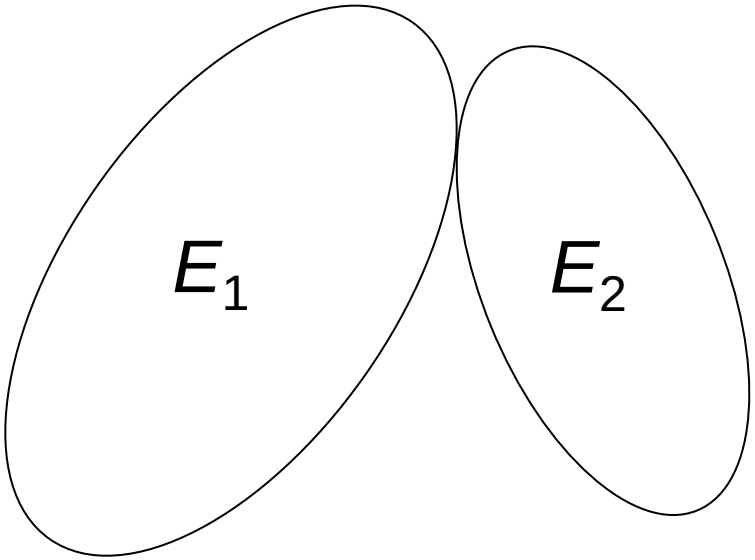
Single-Sided Expansion Factor

- Neither optimistic nor conservative
- May be used to optimise space?



Yi-King Choi (YKC)

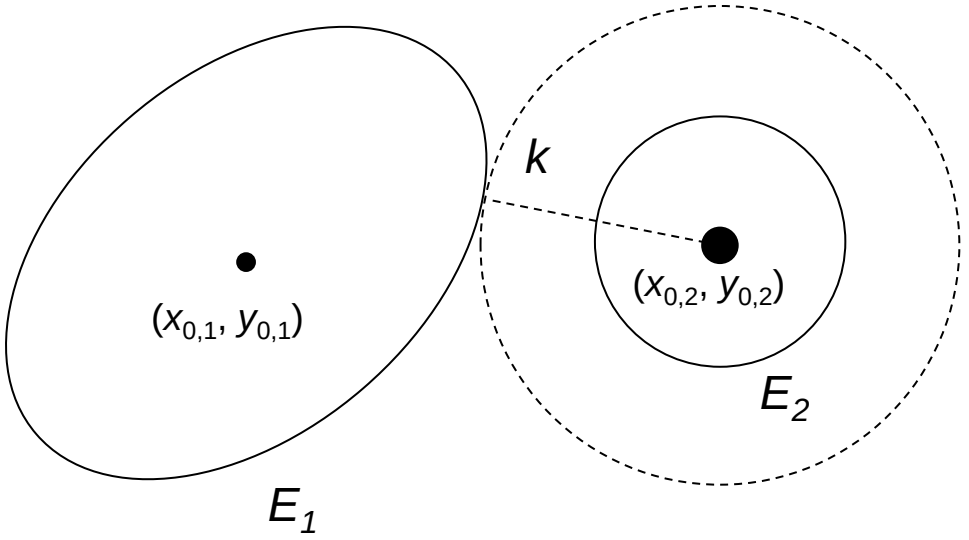
$$P(\lambda) = \det[\lambda \underline{E}_1 - \underline{E}_2(k^2)] = 0$$



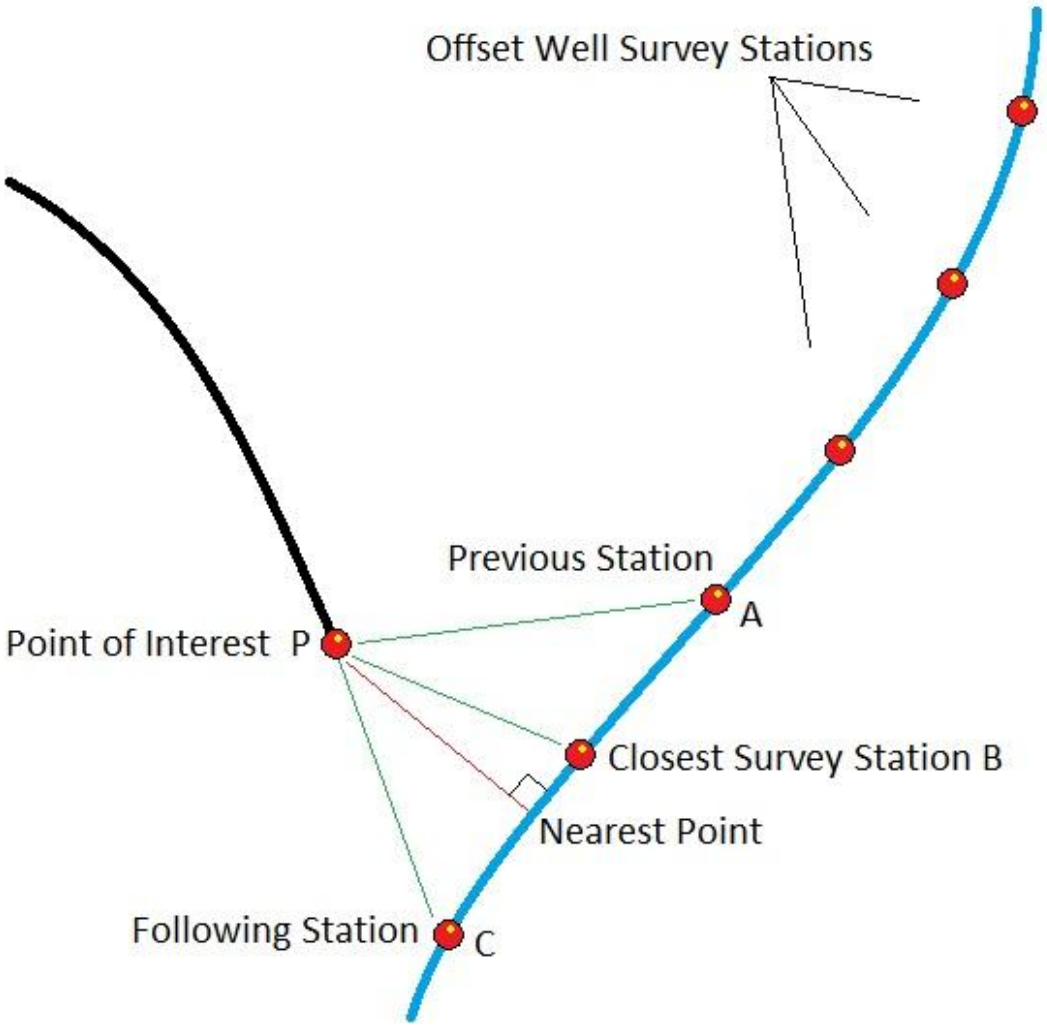
- Robotics studies - PhD Thesis, University of Hong-Kong, 2008
- P(λ) is a cubic equation in λ
- The cubic's discriminant vanishes when the ellipses touch
- Then leads to a quartic equation in the square of the expansion factor k^2
- Quick look methods

Closest Point to an Ellipse

- Toolkit example
- Geometrically equivalent to the single sided expansion of a circle against an ellipse



Scanning Algorithm



- In practice ellipses with high aspect ratios are avoided
- Confers stability to the calculations

Implementation

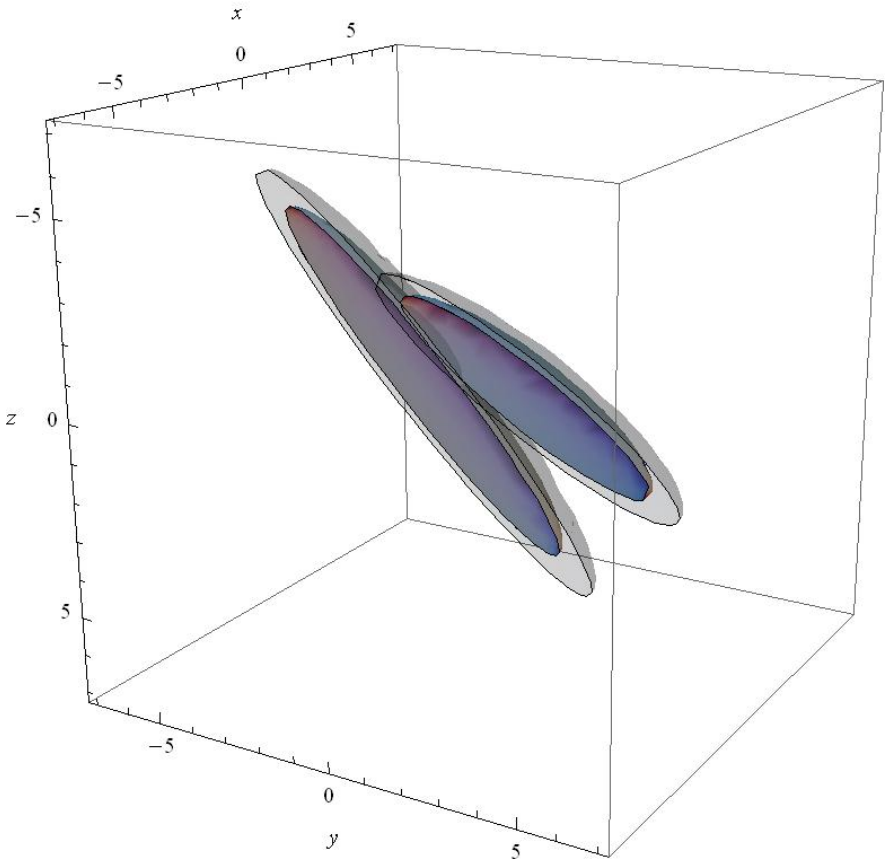
- **Easy to implement**
  - ZPM provides code for 2D and 3D cases**
  - YKC uses a similar framework
  - Test cases provided
- **Execution speed is maintained (or enhanced)**

Method	Visual Basic Real [sec]	Proprietary Application* Imaginary [sec]
PCM	1.0	-
ZPM	1.3	14.1

Time taken for 10^5 calculations

Two-Sided Expansion of Ellipsoids

- Addresses special end condition
- Iterative solution based on other ZPM work
- Used infrequently



Summary

- **Explicit calculation of expansion factors for collision avoidance between two coplanar ellipses is now possible.**
  - Full details of the algorithms will be presented in the paper SPE 159840 at the ATCE, 8th – 10th October 2012, San Antonio.
  - Like-for-like replacement of existing methods
  - Satisfies both geometrical and probabilistic constraints
  - Neither pessimistic nor optimistic
  - Maintains or enhances computational efficiency
- **Provided as a toolkit**
- **Offered for consideration as a replacement industry standard**