Statistical Methods for Calculating the Risk of Collision between Petroleum Wells

Bjørn Erik Loeng and Erik Nyrnes, Statoil ASA

2013-01-24

Presentation Overview

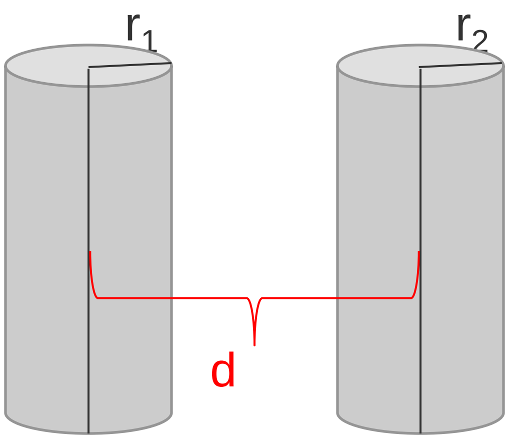
- Introduction of the Problem
- Assumptions
- Current Approach: Separation Factor (Hypothesis Test)
- Statistical Distributions of the Shortest Distance
- Hypothesis Test or Collision Probability?
- Two Points or Several Points?
- Concluding Remarks

Introduction of the Problem

What is the risk of collision between two petroleum wells?

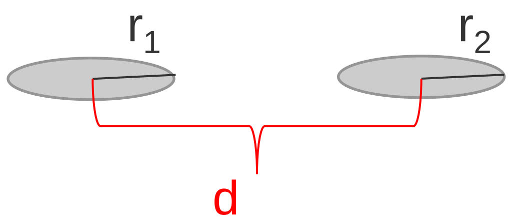
Introduction of the Problem

Collision when $d \leq r_1 + r_2$



Introduction of the Problem

Collision when d ≤ $r_1 + r_2$



$$\mathbf{p}_{\boldsymbol{a}} = [N_a\ E_a\ V_a] \qquad \mathbf{p}_{\boldsymbol{b}} = [N_b\ E_b\ V_b]$$

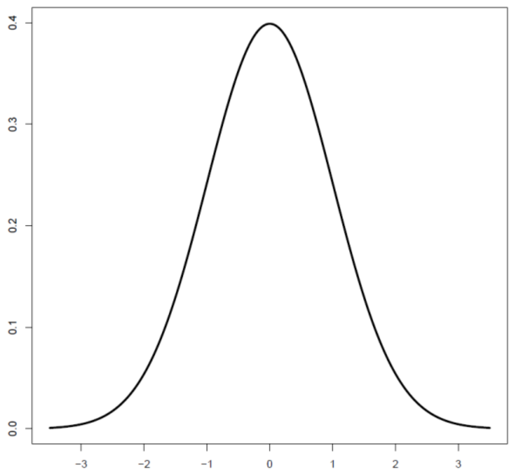
Statoil

Assumptions

All wellbore directional measurements are associated with uncertainty:

$$X \in \{G_x, G_y, G_z, B_x, B_y, B_z, D\}$$

$$X \sim \mathrm{N}(\mu, \sigma^2)$$



Assumptions

- Errors in the directional measurements propagate into the NEV coordinates of the two closest points in the reference well and the offset well
- Errors in the NEV coordinates propagate into the shortest distance *d* between the two closest points

$$X \in \left\{G_x, G_y, G_z, B_x, B_y, B_z, D\right\} \quad X \sim \mathrm{N}(\mu, \sigma^2)$$

$$\mathbf{p} = \left[N_a\ E_a\ V_a\ N_b\ E_b\ V_b\right] \sim \mathrm{N}_6(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

$$d = \sqrt{(N_a - N_b)^2 + (E_a - E_b)^2 + (V_a - V_b)^2} \sim \mathrm{N}(\mu, \sigma^2)$$

Statoil

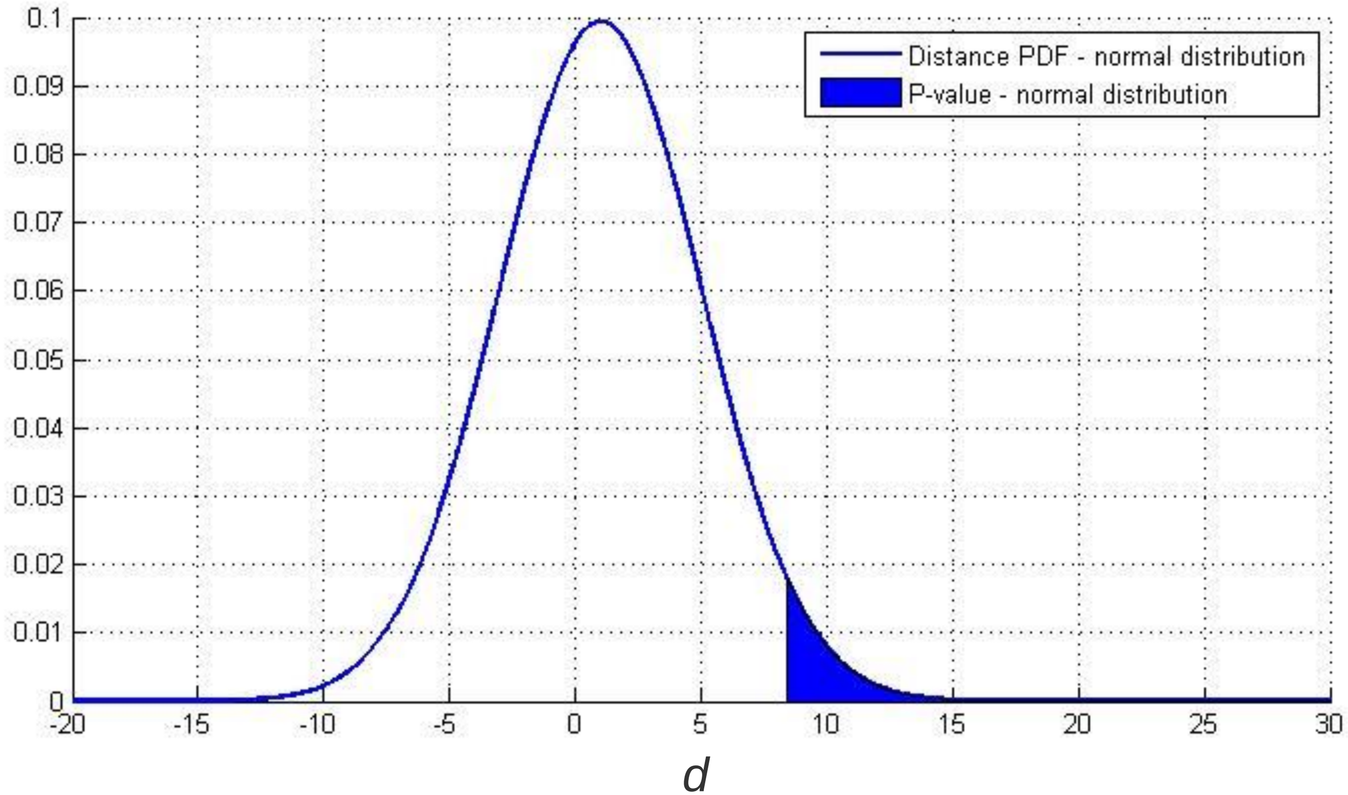
Current Approach: Separation Factor

- Statoil's way of defining the separation factor:

$$SF = \frac{d - (r_1 + r_2)}{k_\alpha \sigma_d}$$

- The SF criterion can be formulated as a statistical hypothesis test derived from a standard normally distributed test statistic
- We accept the risk of collision if SF > 1
- k_α: critical value of Z(0,1) for a given α
- σ_d: standard deviation of *d*

Separation Factor (Hypothesis Test)



Expected collision: Expected distance is 1 meter

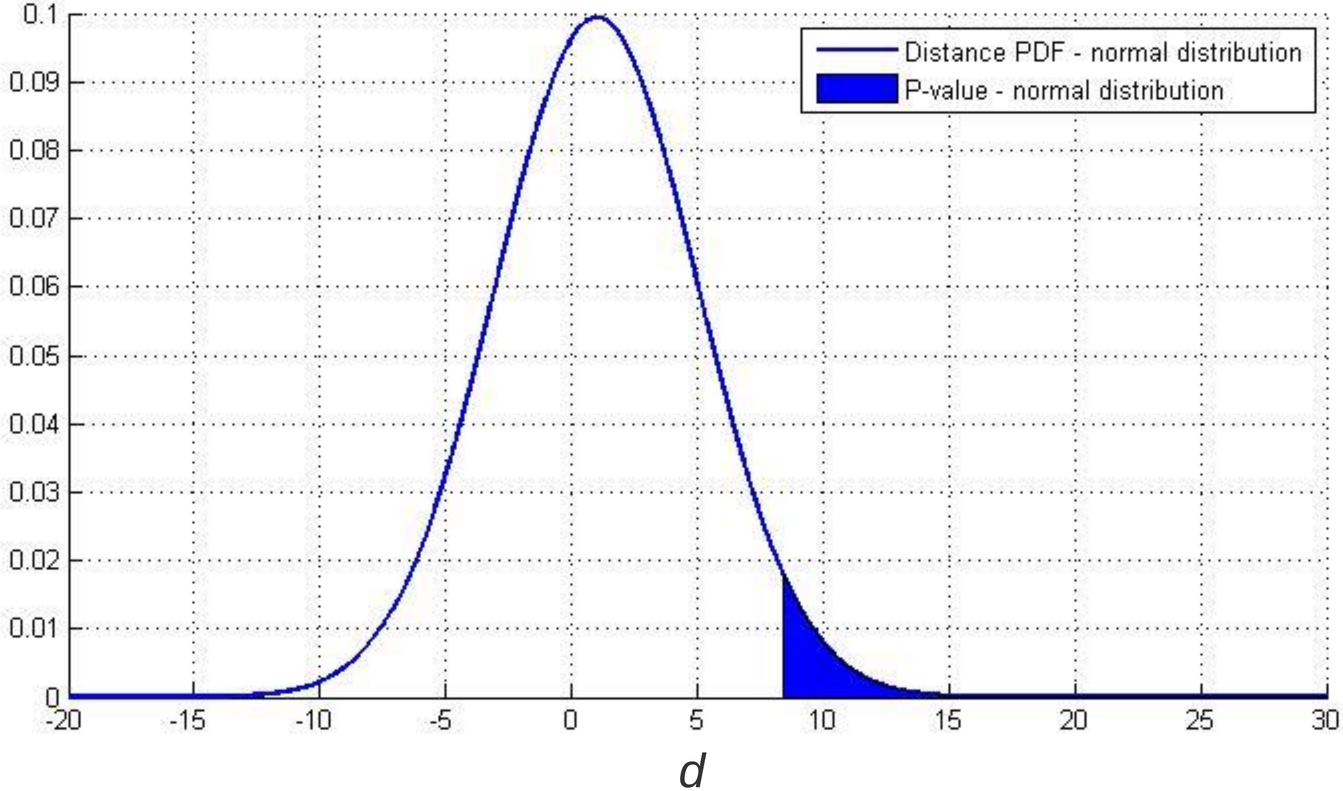
Planned distance is about 8 meters

We accept the risk of collision if SF > 1, that is if the **p-value**

P(we plan/measure ≥ the planned/measured distance when there actually will be a collision) ≤ α

Statoil

Separation Factor (Hypothesis Test)



Expected collision: Expected distance is 1 meter

Planned distance is about 8 meters

Small p-value implies large separation factor

Statistical Distribution of the Shortest Distance

The errors in the NEV coordinates propagate into in the shortest distance between the reference well and the offset well.

$$\mathbf{p} = [N_a \; E_a \; V_a \; N_b \; E_b \; V_b] \sim \mathrm{N}_6(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

$$d = \sqrt{(N_a - N_b)^2 + (E_a - E_b)^2 + (V_a - V_b)^2} \sim N(\mu, \sigma^2)$$

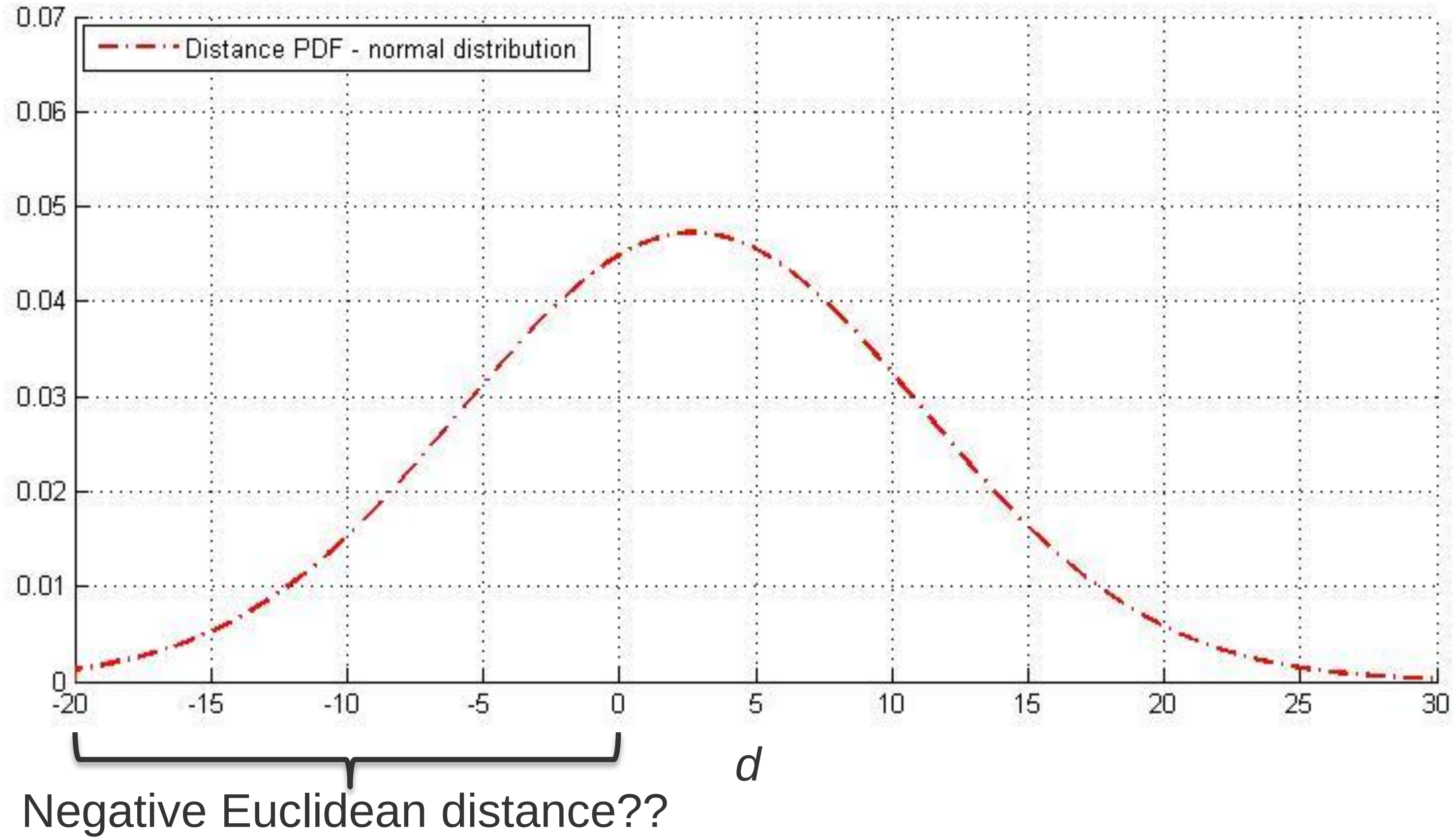
Statistical Distribution of the Shortest Distance

The errors in the NEV coordinates propagate into in the shortest distance between the reference well and the offset well.

$$\mathbf{p} = \left[N_a\ E_a\ V_a\ N_b\ E_b\ V_b\right] \sim \mathrm{N}_6(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

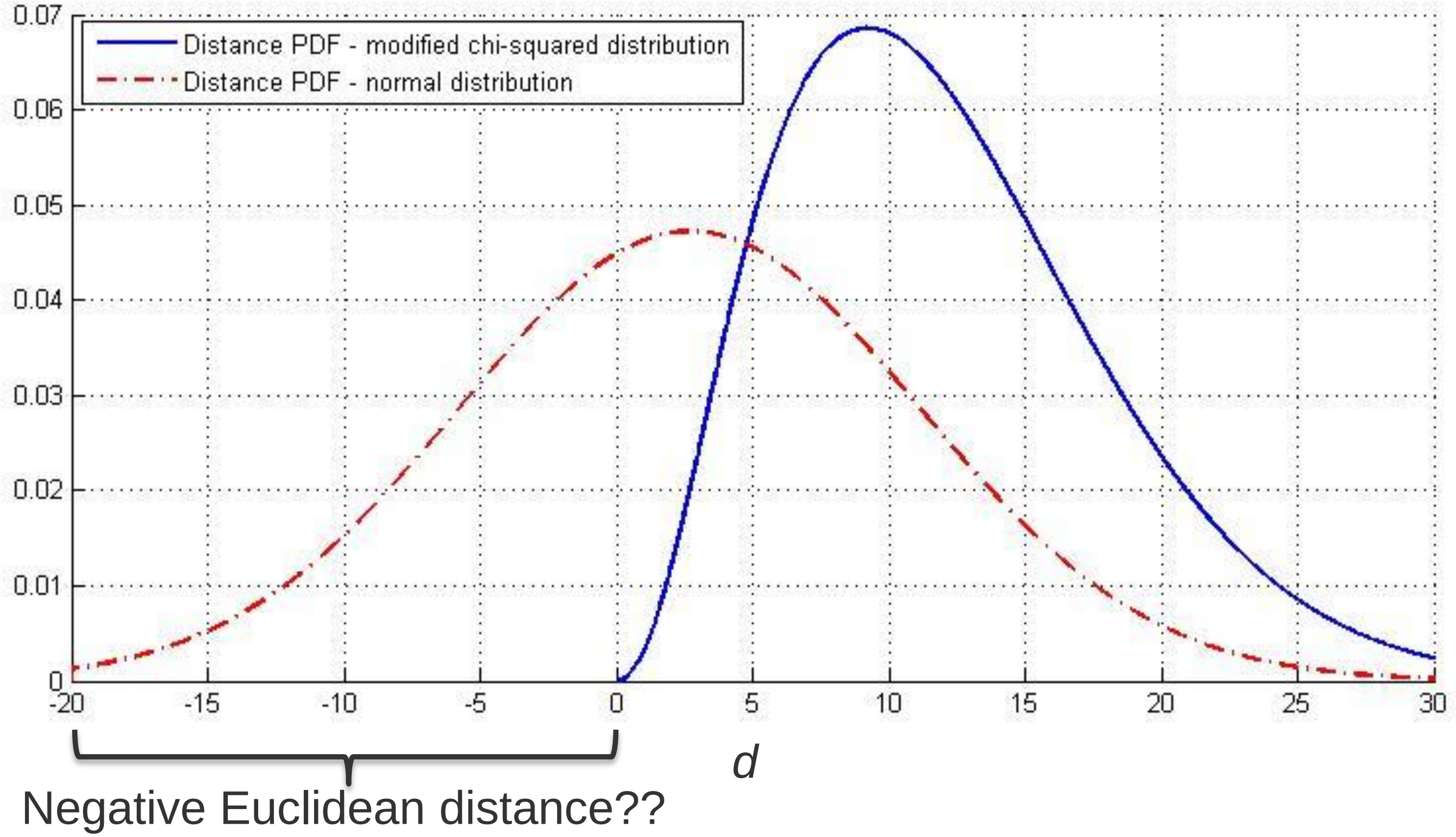
$$d = \sqrt{(N_a - N_b)^2 + (E_a - E_b)^2 + (V_a - V_b)^2} \sim \quad ?$$

Statistical Distribution of the Shortest Distance



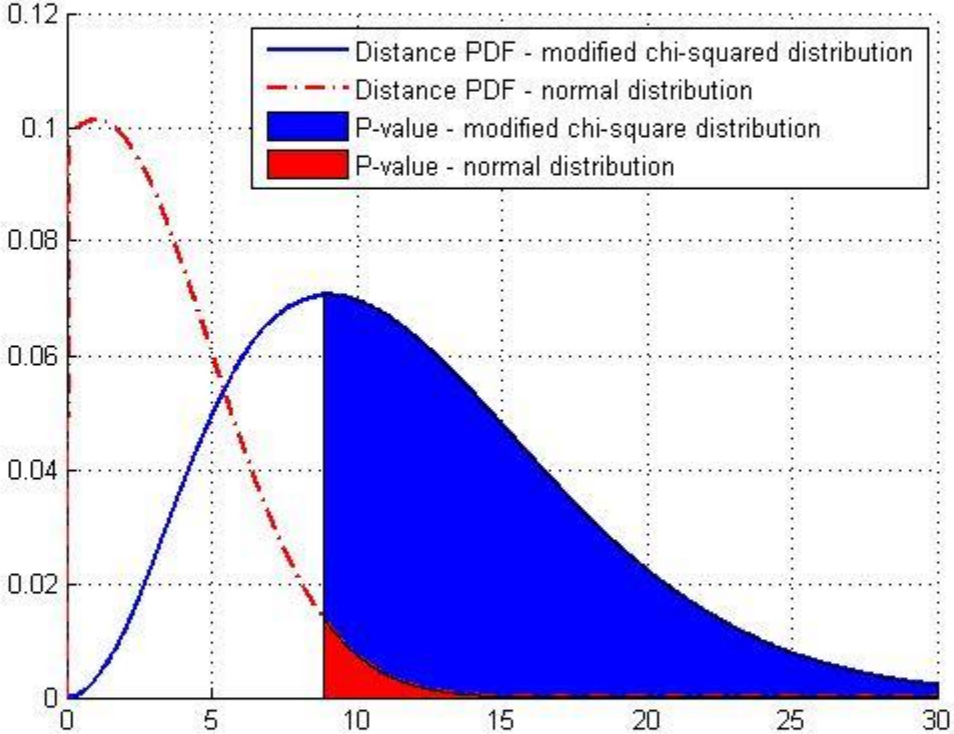
Statoil

Statistical Distribution of the Shortest Distance



Statoil

Hypothesis Test for a Realistic Well Pair



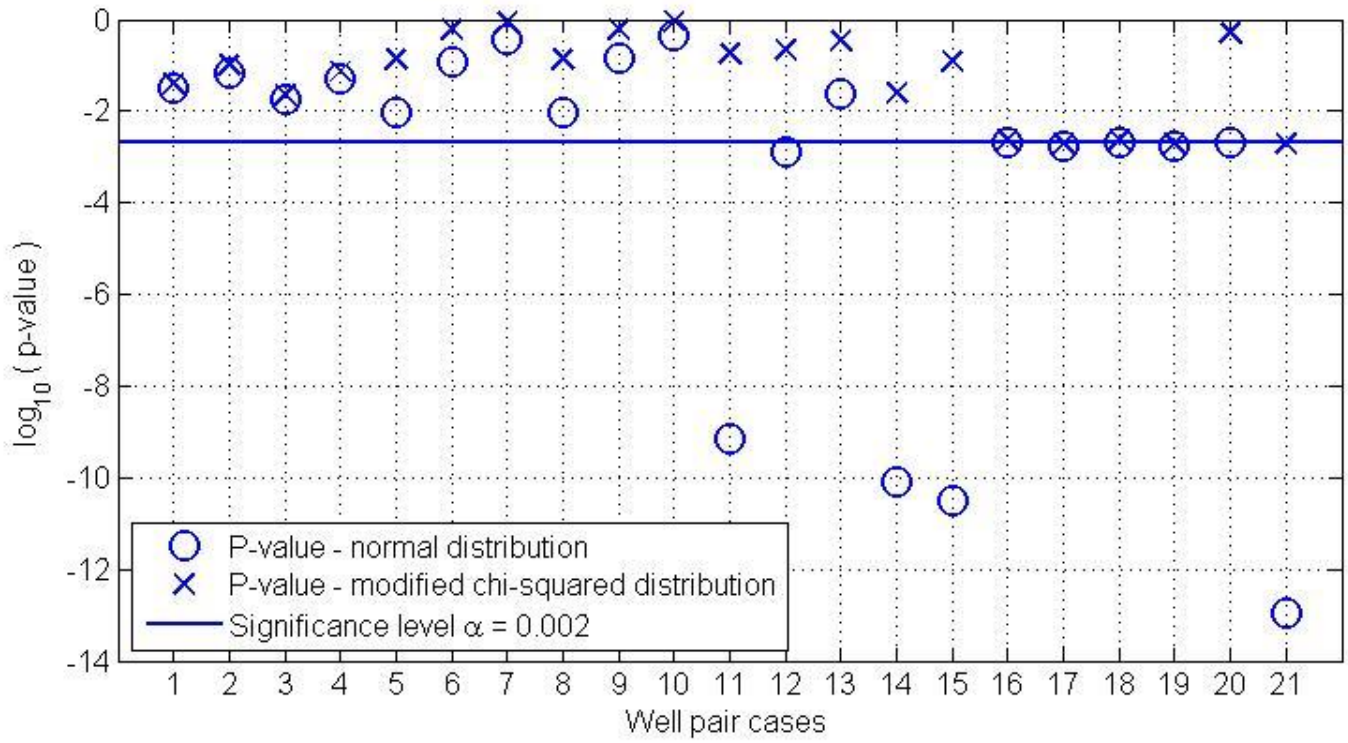
d

Expected collision: Expected distance is 1 meter

Planned distance is about 8 meters

The modified χ^2 distribution is more conservative than the normal distribution

Hypothesis Test Results



The modified χ^2 distribution is more conservative than the normal distribution

Statoil

Hypothesis Test

Hypothesis Test or Collision Probability?

Hypothesis Test or Collision Probability?

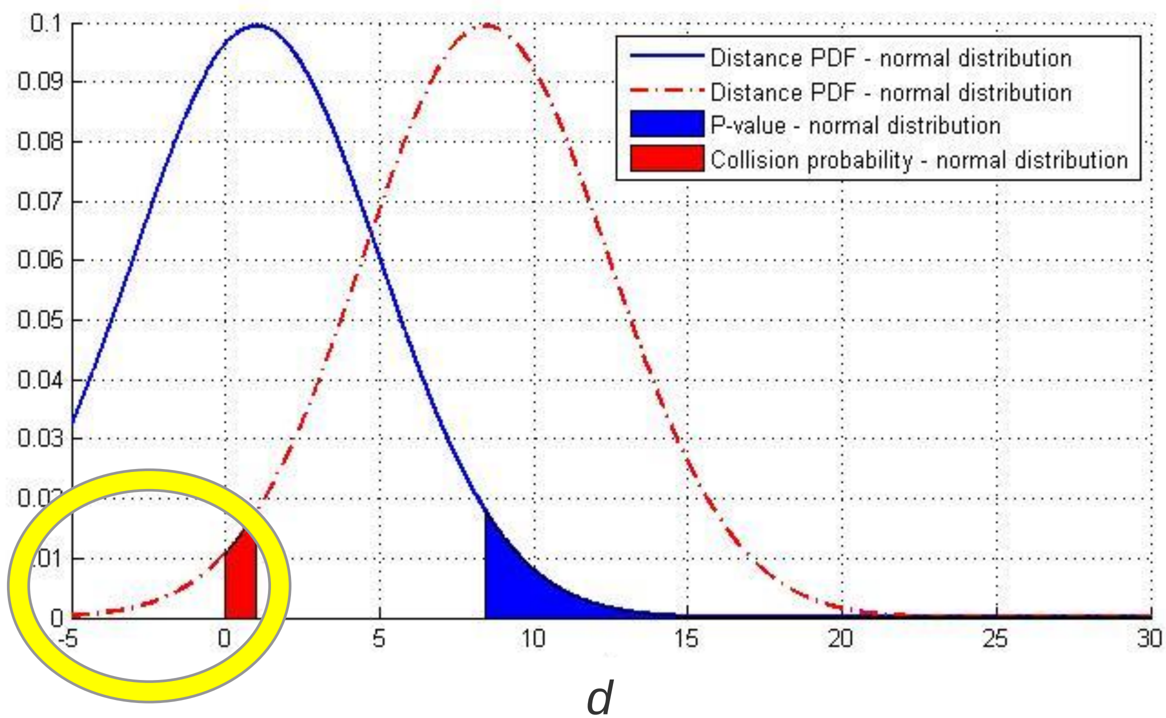
- **Hypothesis test:** We evaluate the risk of collision based on the requirement

P (we conclude that there will not be a collision when it actually will be a collision) $\leq \alpha$

- **Collision probability:** We accept the risk of collision if the collision probability

P (there will be a collision) $\leq \beta$

Hypothesis Test or Collision Probability?



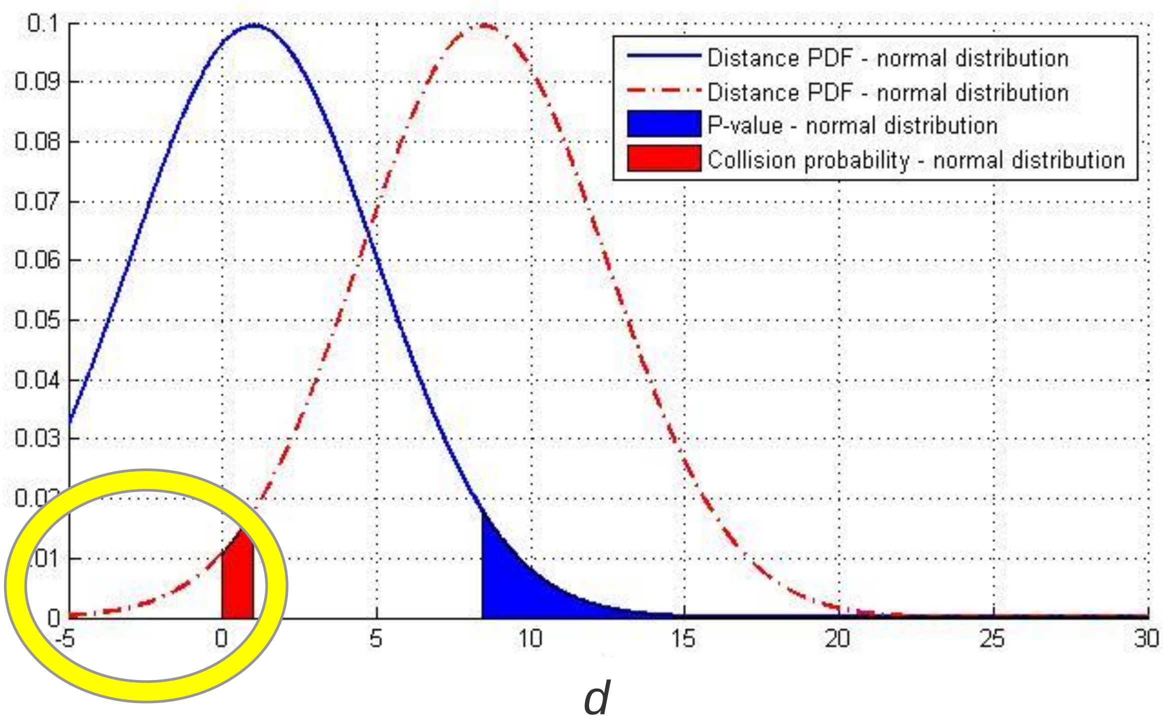
For a collision, distance is 1 meter

Planned distance is about 8 meters

For a normally distributed distance *d* the p-value criterion is more conservative than collision probability criterion when α = β

Statoil

Hypothesis Test or Collision Probability?



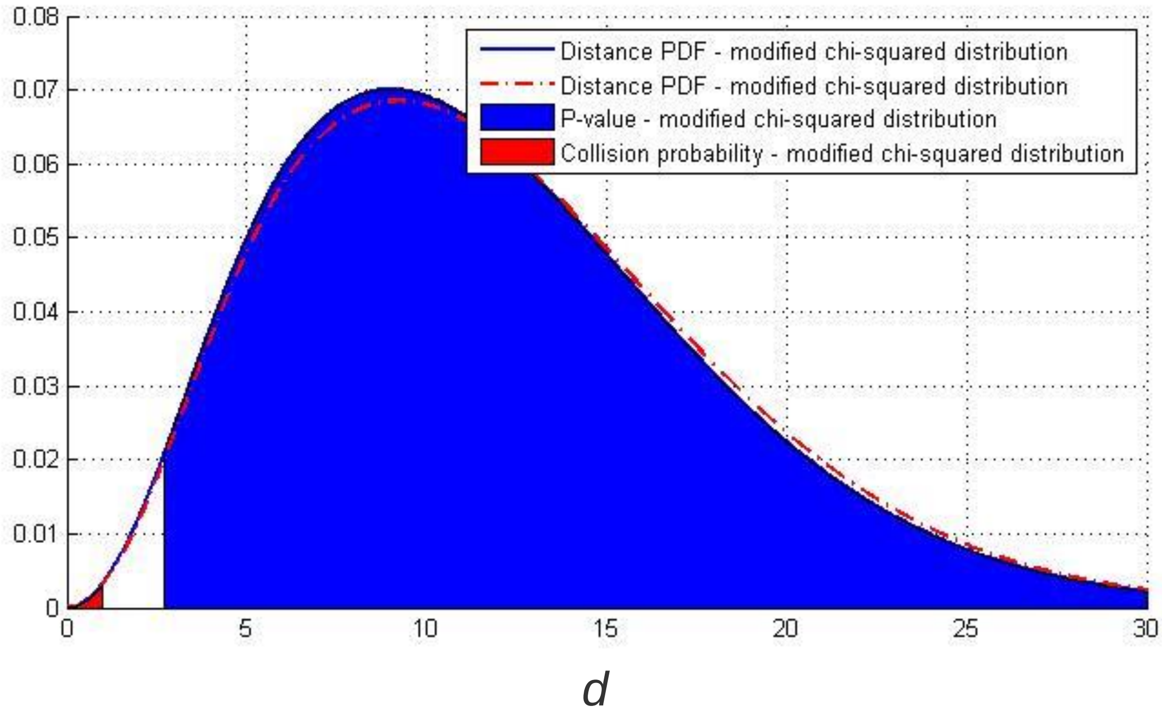
For a collision, distance is 1 meter

Planned distance is about 8 meters

For a normally distributed distance *d* the collision probability is difficult to interpret because of the negative values

Statoil

Hypothesis Test or Collision Probability?



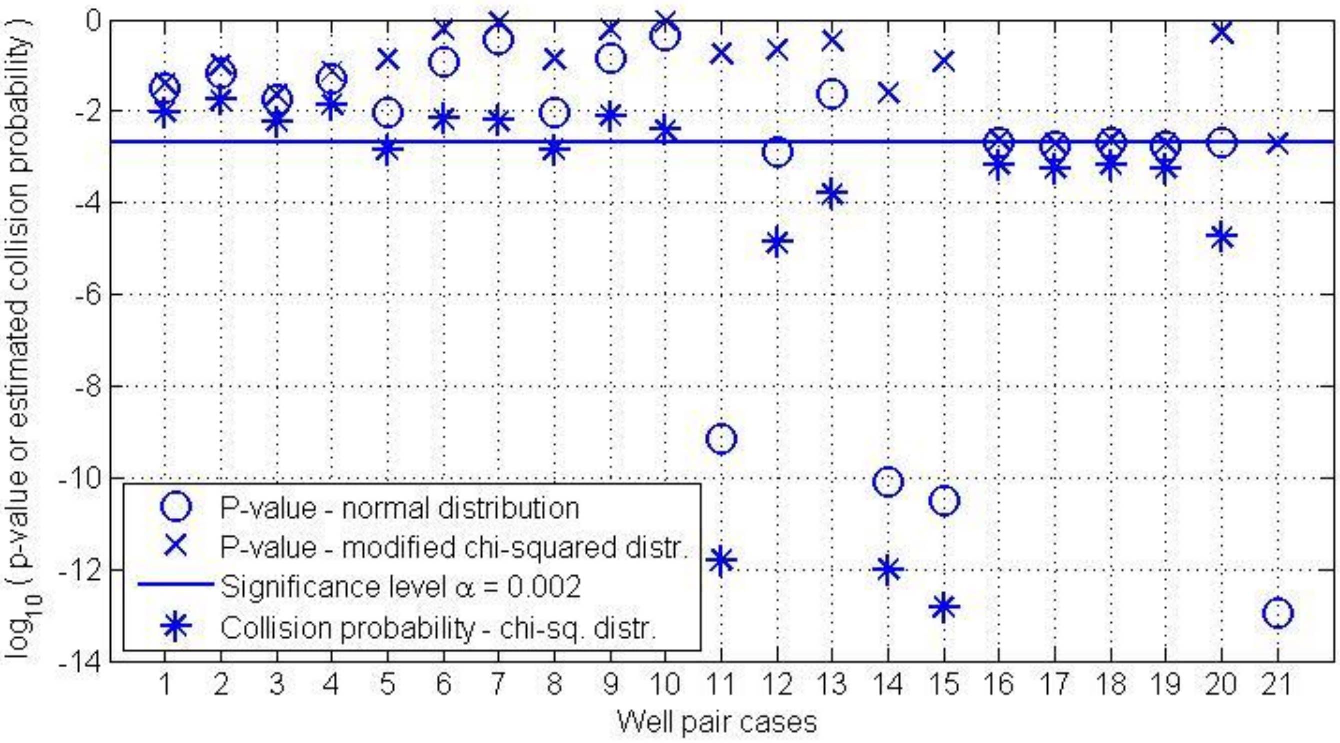
For a collision, distance is 1 meter

Planned distance is about 3 meters

For a modified χ^2 distributed distance *d* the p-value criterion is more conservative than the collision probability criterion when $\alpha = \beta$

Statoil

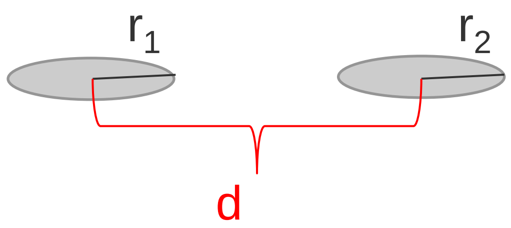
Hypothesis Test or Collision Probability?



The estimated collision probability is smaller than both p-values

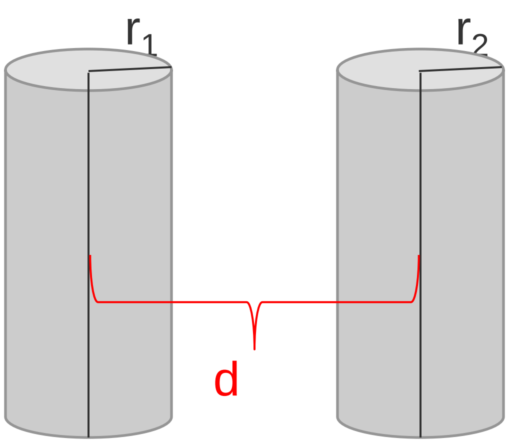
Two Points or Several Points?

Collision when $d \leq r_1 + r_2$



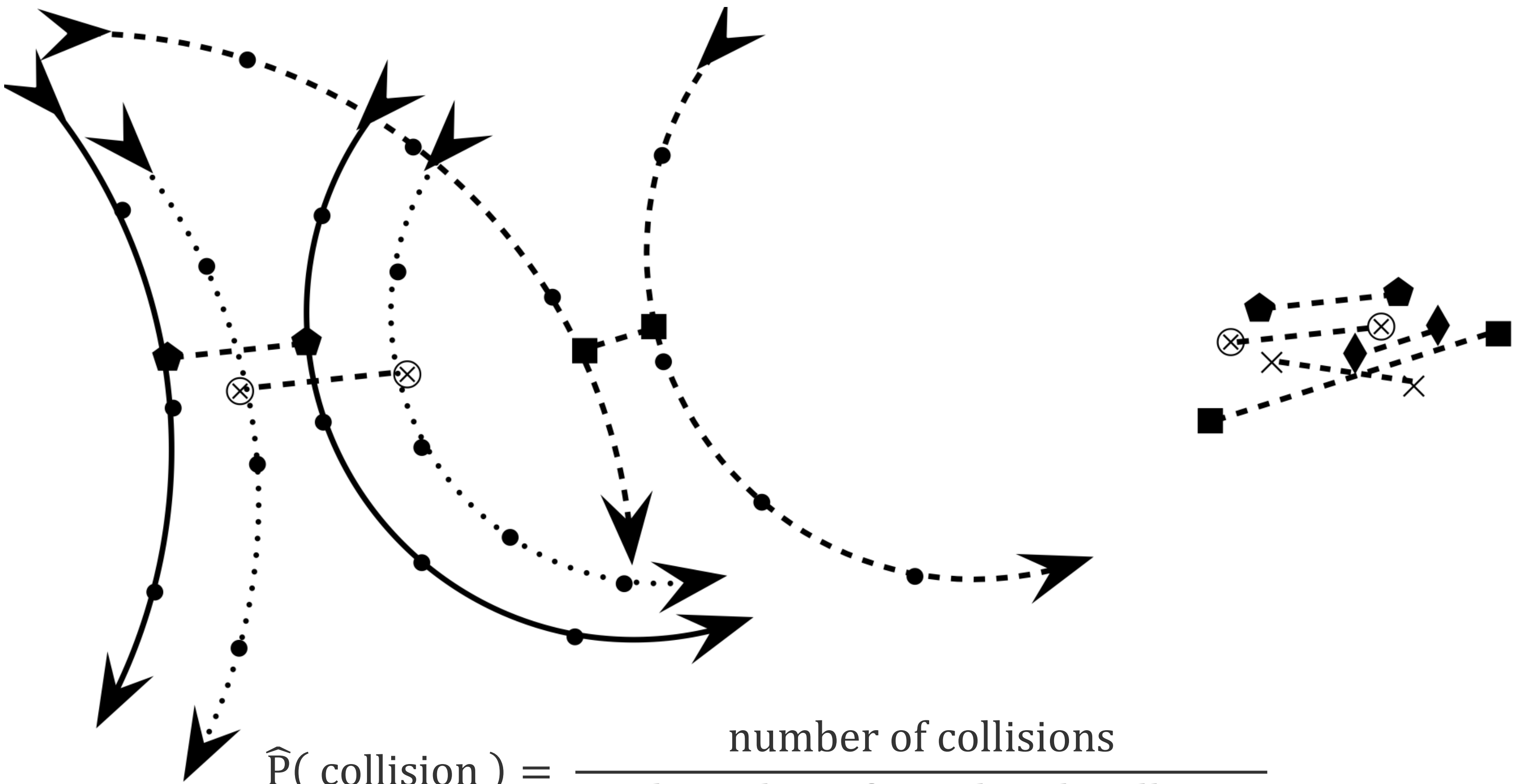
Two Points or Several Points?

Collision when $d \leq r_1 + r_2$



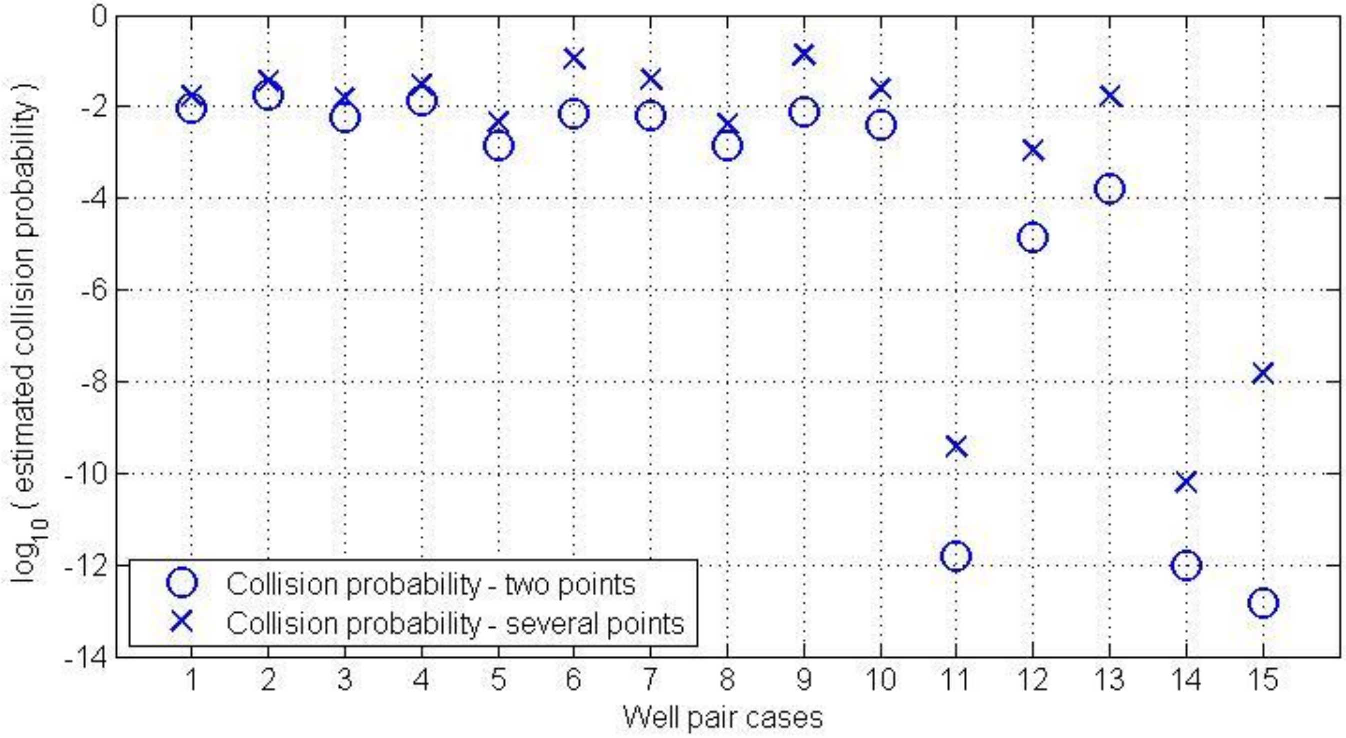
Statoil

Monte Carlo Simulation Method



$$\widehat{P}(\text{ collision }) = \frac{\text{number of collisions}}{\text{total number of simulated well pairs}}$$

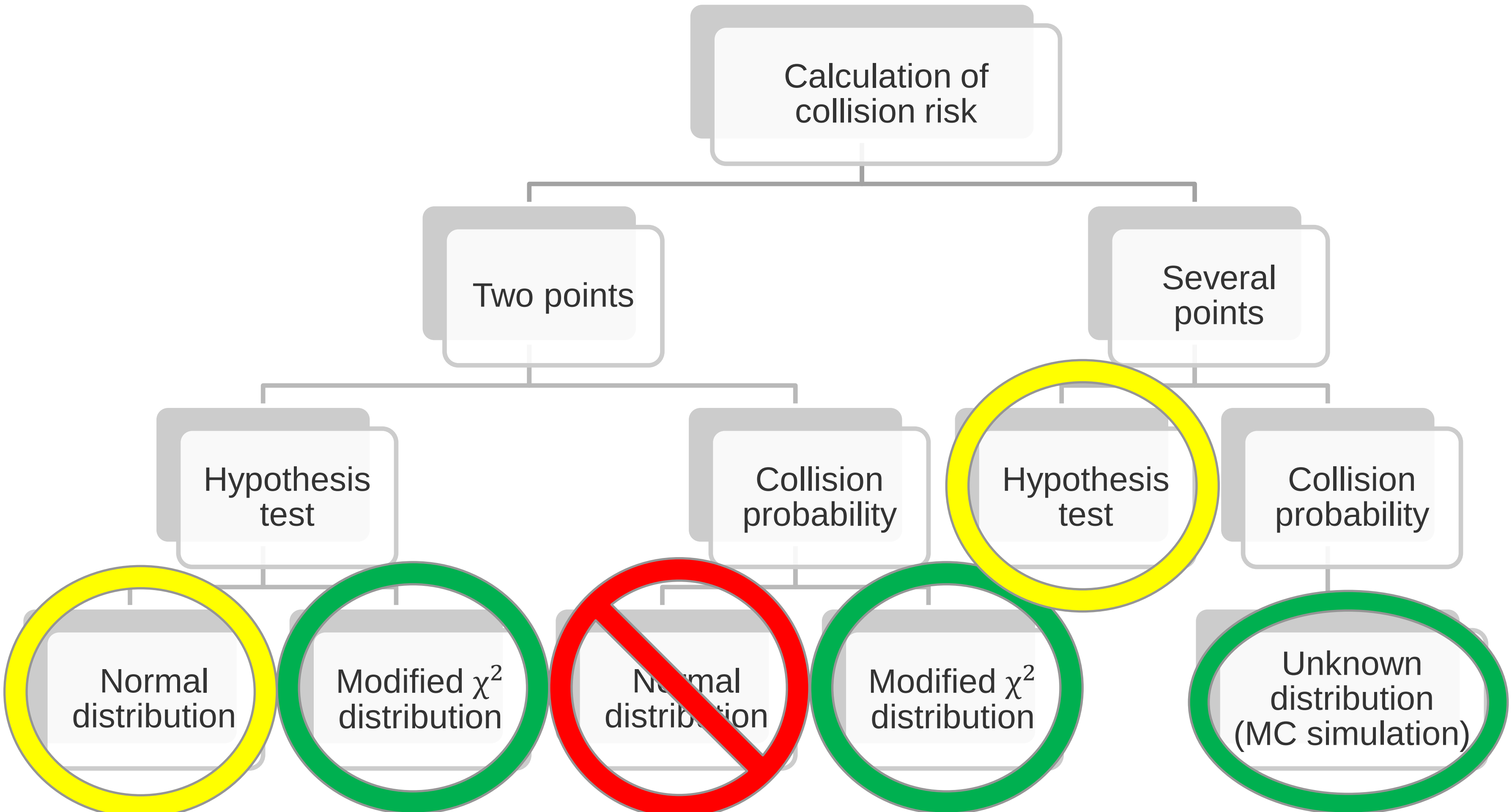
Two Points or Several Points?



The estimated collision probability is greater when taking several points (well segments) into account

Statoil

Concluding Remarks



Concluding Remarks

- The modified χ^2 distribution is more accurate than the normal distribution when considering the Euclidean distance between two points
- For a hypothesis test, the normal distribution gives less conservative results than the modified χ^2 distribution
- The collision probability is difficult to interpret with the normal distribution, while it is a simple task using the modified χ^2 distribution
- Estimated collision probability tends to be smaller than the p-values for both the modified χ^2 distribution test and the normal distribution test
- Taking into account more points than only the two closest points will increase the estimated collision risk significantly

References

- Loeng (2012): Statistical Methods for Calculating the Risk of Collision Between Petroleum Wells. *MSc Thesis, Norwegian University of Science and Technology.*
- Gjerde, Eidsvik, Nyrnes, Bruun (2011): Positioning and Position Errors of Petroleum Wells. *Journal of Geodetic Science, 1(2): 158-160.*
- Sheil and O'Muircheartigh (1977): Algorithm as 106: The Distribution of Non-Negative Quadratic Forms in Normal Variables. *Journal of the Royal Statistical Society. Series C (Applied Statistics), 26(1): 92-98.*
- Williamson (2000): Accuracy Prediction for Directional Measurement While Drilling. *SPE Drilling & Completion, 15(4): 221-223.*
- Rubinstein and Kroese (2004): The Cross-Entropy Method: A Unified Approach to Combinatorial Optimization, Monte-Carlo Simulation, and Machine Learning. *Springer.*

There's never been a better time for good ideas

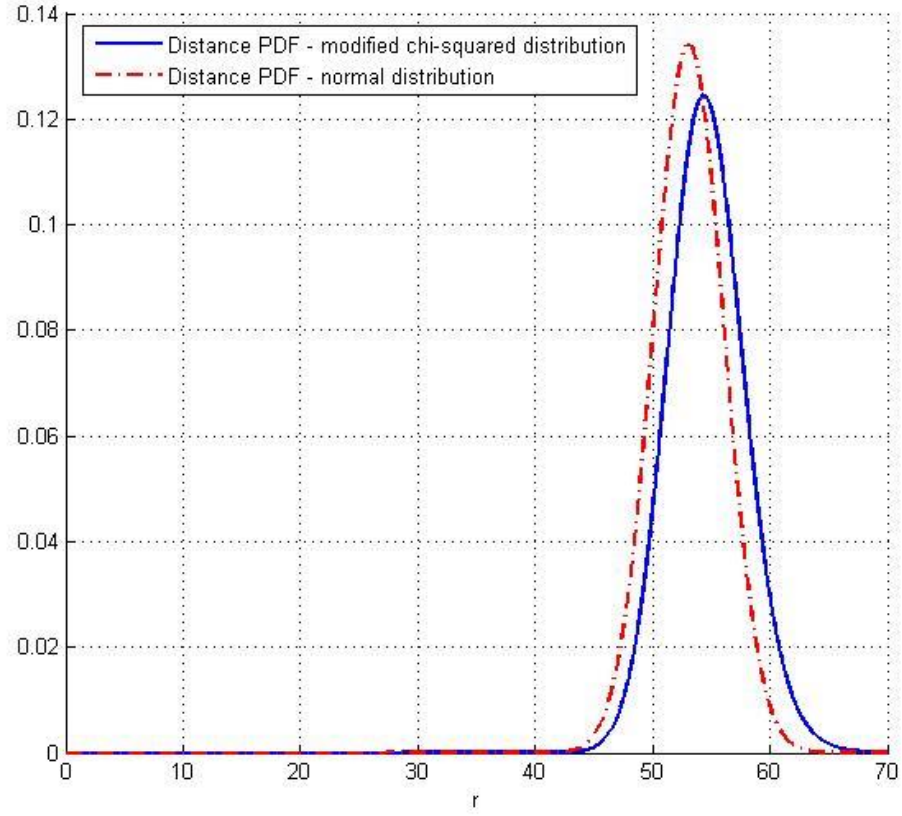
Statistical Methods For Calculating the Risk of Collision Between Petroleum Wells

Bjørn Erik Loeng
TPD D&W DWT DT DD
bloe@statoil.com
Tel: +4797629440

www.statoil.com

Extra Material

Statistical Distribution of the Shortest Distance

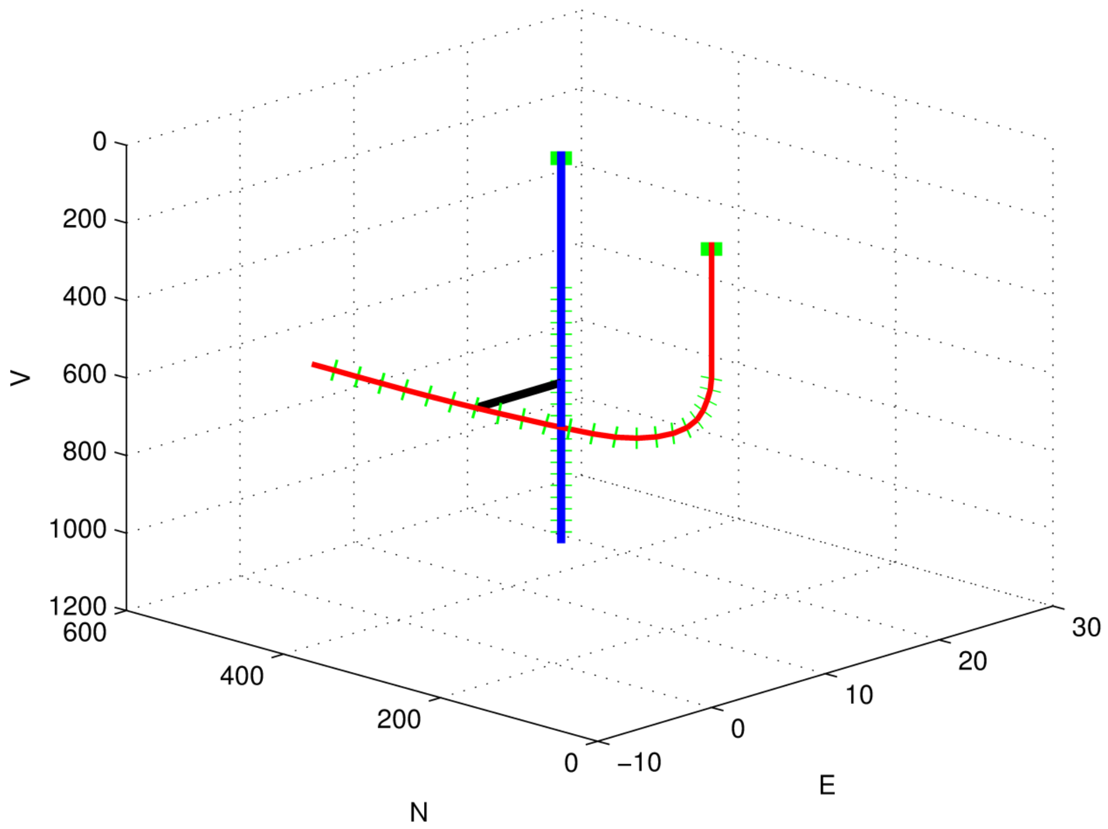


Hypothesis Test or Collision Probability?

$$\mathbf{p} = [N_a \; E_a \; V_a \; N_b \; E_b \; V_b] \sim \mathrm{N}_6(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

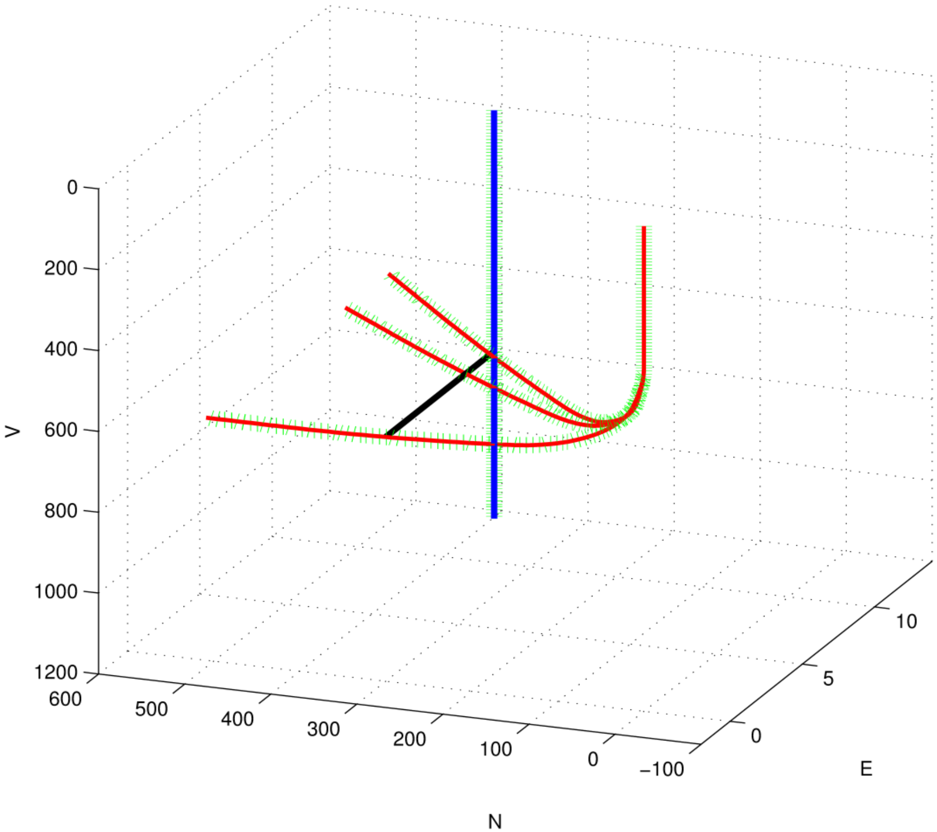
- Hypothesis test:
  - ***p***: assumed (measured or planned) positions of wells
  - ***μ***: true (unknown) positions of wells
- Collision probability:
  - ***p***: true (unknown) positions of wells
  - ***μ***: assumed (measured or planned) positions of wells

Variance-Reducing Methods for Rare Events



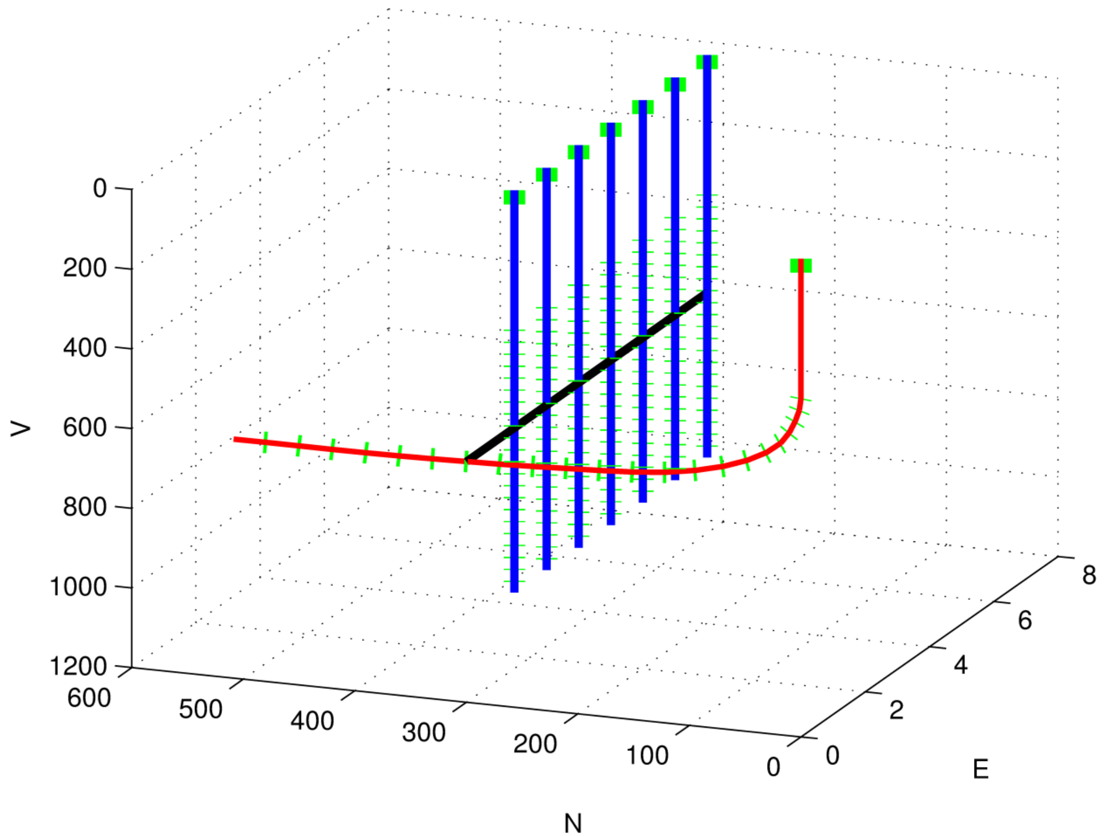
Simulating segments of wells (several points) requires more computational power but there exist variance-reducing methods that reduce the computing time

The Cross-Entropy Method



Statoil

The Enhanced Monte Carlo Method



Statoil's Collision Avoidance Criteria

Reference and Offset Wells

- Consider two points, one in the reference well and one in the offset well, with position vectors *u* and *v* respectively:

$$u = \begin{pmatrix} N \\ E \\ V \end{pmatrix} \qquad v = \begin{pmatrix} N \\ E \\ V \end{pmatrix}$$

- $Cov(u) = \Sigma_u$ and $Cov(v) = \Sigma_v$:

$$\Sigma_u = \begin{pmatrix} \sigma^2_{NN} & \sigma^2_{NE} & \sigma^2_{NV} \\ \sigma^2_{EN} & \sigma^2_{EE} & \sigma^2_{EV} \\ \sigma^2_{VN} & \sigma^2_{VE} & \sigma^2_{VV} \end{pmatrix} \qquad \Sigma_v = \begin{pmatrix} \sigma^2_{NN} & \sigma^2_{NE} & \sigma^2_{NV} \\ \sigma^2_{EN} & \sigma^2_{EE} & \sigma^2_{EV} \\ \sigma^2_{VN} & \sigma^2_{VE} & \sigma^2_{VV} \end{pmatrix}$$

Evaluating the Distance Between Reference and Offset wells

- Distance between the reference well and the offset well:

$$D = \sqrt{(u-v)^T(u-v)}$$

- Is the distance *D* representing any risk?
- Is the distance statistically different from zero?
- One way to evaluate such problems is to apply a statistical hypothesis test
- The hypotheses (or the hypothesis test) for *D* can be formulated by:

H_0: $E(D) = 0$ versus H_A: $E(D) \neq 0$

- If H_0 is true there is a high risk of collision
- If H_0 is false there is a low risk of collision

Required Input Data

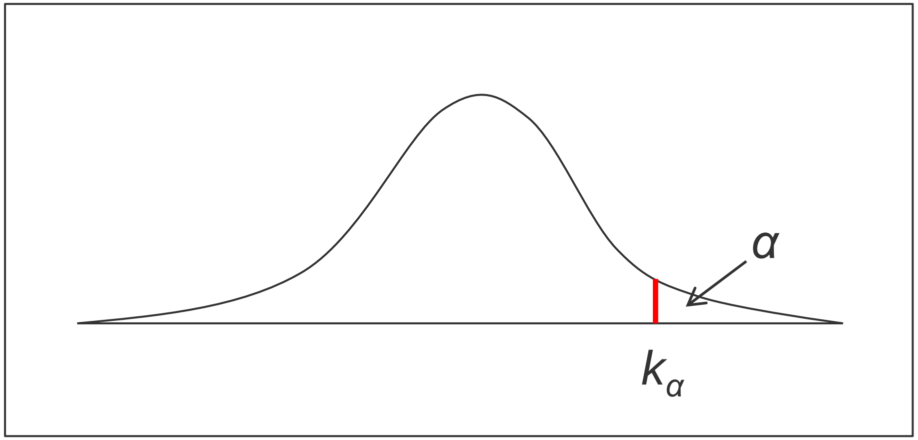
- Two candidate points in the reference well and the offset well
- Covariance matrices of the well positions
- Diameters of the reference well and the offset well
- Test statistic for the hypothesis test
- Significance level of the hypothesis test

Test of Hypotheses

- «Standardization» of *D* gives the test statistic:

$$w = \frac{D}{\sigma_D} \sim N(0,1)$$

- Hypothesis test:
  - Reject H_0 if $w \geq k_\alpha$
  - Accept H_0 if $w < k_\alpha$



where k_α is the $100(1-\alpha)$ percentage quantile of the standard normal distribution $N(0,1)$ for a given significance level α.

Statoil

The Uncertainty σ_D of the Distance *D*

$D^2 = (u - v)^T (u - v)$ *Eq. (1)*

with $cov(u) = \Sigma_u$ and $cov(v) = \Sigma_v$

Differentiation of *Eq. (1)* with respect to u and v gives:

$dD = \frac{(u-v)}{D} d(u - v)$ *Eq. (2)*

Covariance propagation gives:

$\sigma_D^2 = \frac{1}{D^2} (u - v)^T (\Sigma_u + \Sigma_v)(u - v)$ *Eq. (3)*

Derivation of Separation Factor

- Reject H_0 if:

$$w = \frac{D}{\sigma_D} \geq k_\alpha \rightarrow z = \frac{D}{\sigma_D k_\alpha} \geq \frac{k_\alpha}{k_\alpha} \rightarrow z = \frac{D}{k_\alpha \sigma_D} \geq 1 \quad \textit{Eq. (4)}$$

  - Small risk of collision

- Accept H_0 if:

$$z = \frac{D}{k_\alpha \sigma_D} < 1 \quad \textit{Eq. (5)}$$

  - High risk of collision

Separation Factor – General Formulation

$$SF = \frac{D - \frac{d_1 + d_2}{2}}{k_\alpha \sigma_D}$$

D = 3D centre-centre distance between the reference and the offset wells

d_1, d_2 = wellbore diameters (casing or open-hole diameter at the points of interest)
σ_D = standard deviation of *D*
k_α = critical value of *N(0,1)* for a given α

Statoil

Separation Factor – Statoil's Version

- Basic assumptions:

  $\text{Cov}(u, v) = 0$

  $D \sim N(\mu, \sigma^2)$

  $\alpha = \frac{1}{500} \rightarrow k_\alpha = 2.878$

- The SF formula used by Statoil:

$$SF = \frac{D - \frac{d_1 - d_2}{2}}{2.878\,\sigma_D}$$

Reference

- Other types of hypothesis tests are described and suggested by Tony Gjerde in his Master's thesis (2008):

  "A heavy tailed statistical model applied in anti-collision calculations for petroleum wells"

- This thesis also presents interesting information regarding the normality assumption for the distance between the reference and offset wells
- See also papers by e.g. J. Thorogood, H. Williamson, A. Brooks, etc.

Concluding Remarks

- The use of separation factor may lead to different level of collision avoidance decisions depending on the input parameters being used
- Collision avoidance decisions can be taken without considering the size, direction and the position of the error ellipses of the points of interest in the offset and the reference well
- What needs to be considered is the position coordinates of the two points of interest, their covariance matrices and the statistical significance of the distance between them
- Could the significance level be adjusted to match a desired probability of well collision?