When is it Valid to Assign a Tighter Error Model to MSA-Processed Data?

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A Priori and Target Error Models

A Priori Error Model

Weighting Function	Basic model	Prop. Mode
Sensors		
ABX	0.0004 g	S
ABY	0.0004 g	S
ABZ	0.0004 g	S
ASX	0.0005	S
ASY	0.0005	S
ASZ	0.0005	S
MBX	70 nT	S
MBY	70 nT	S
MBZ	70 nT	S
MSX	0.0016	S
MSY	0.0016	S
MSZ	0.0016	S
AMI	150 nT	S

⋮

Target Error Model

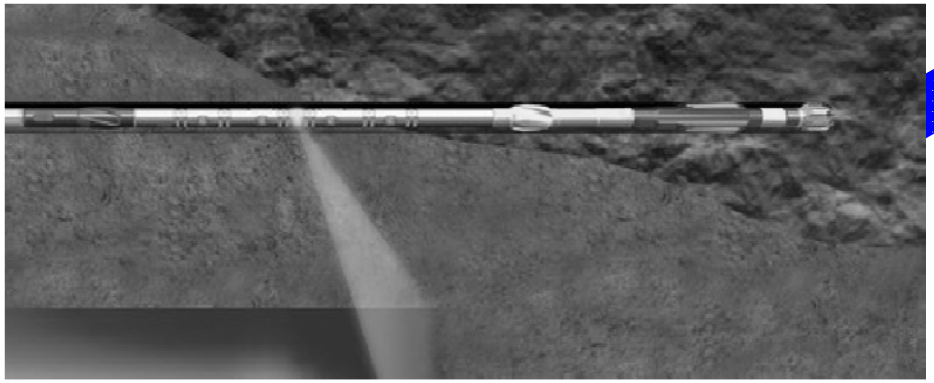
Weighting Function	Basic model	Prop. Mode
Sensors		
ABX	0.0004 g	S
ABY	0.0004 g	S
ABZ	0.0004 g	S
ASX	0.0005	S
ASY	0.0005	S
ASZ	0.0005	S
MBX	~~70 nT~~ 35 nT	S
MBY	~~70 nT~~ 35 nT	S
MBZ	~~70 nT~~ 35 nT	S
MSX	~~0.0016~~ 0.0008	S
MSY	~~0.0016~~ 0.0008	S
MSZ	~~0.0016~~ 0.0008	S
AMI	150 nT	S

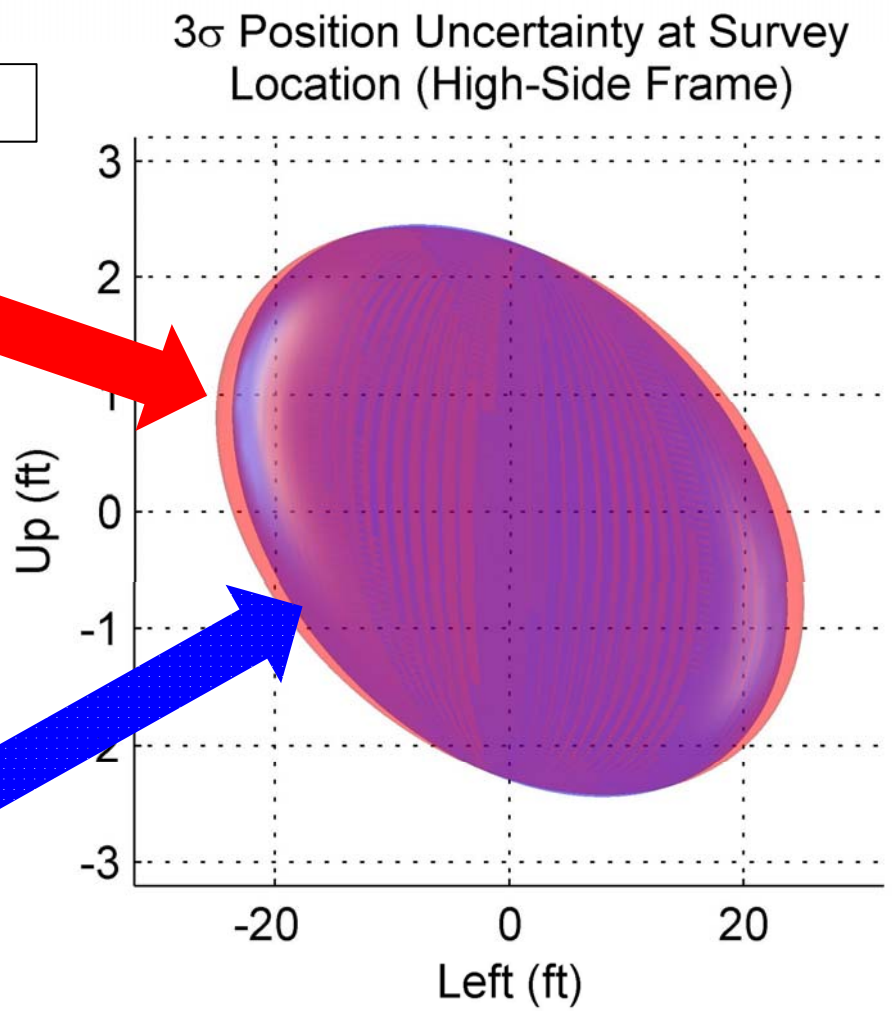
⋮

What Defines a Valid Error Model?

Weighting Function	Basic model	Prop. Mode
Sensors		
ABX	0.0004 g	
ABY	0.0004 g	S
ABZ	0.0004 g	S
ASX	0.0005	S
ASY	0.0005	S
ASZ	0.0005	S
MBX	70 nT	S
MBY	70 nT	S
MBZ	70 nT	S
MSX	0.0016	S
MSY	0.0016	S
MSZ	0.0016	S

A Priori Error Model

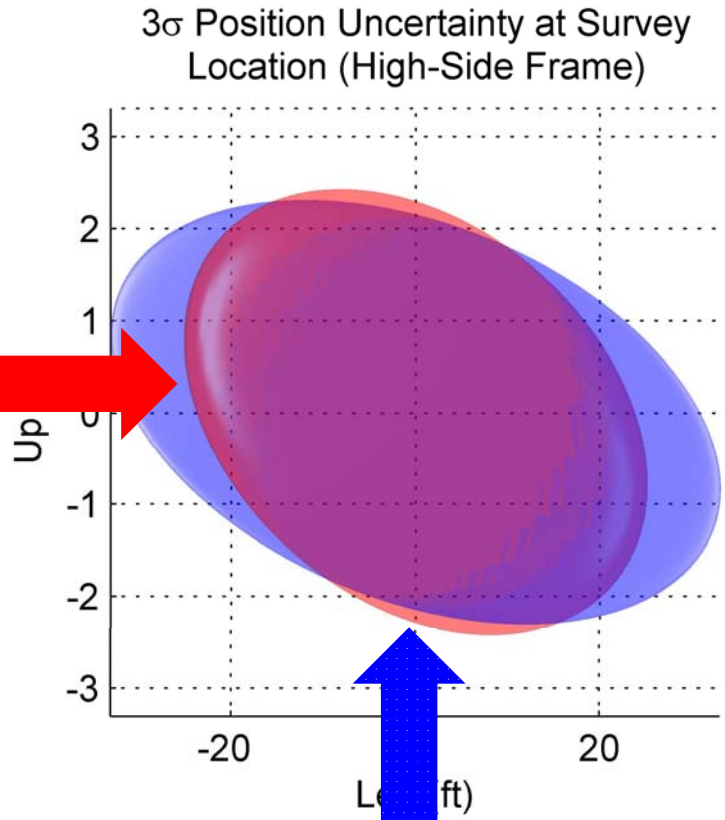


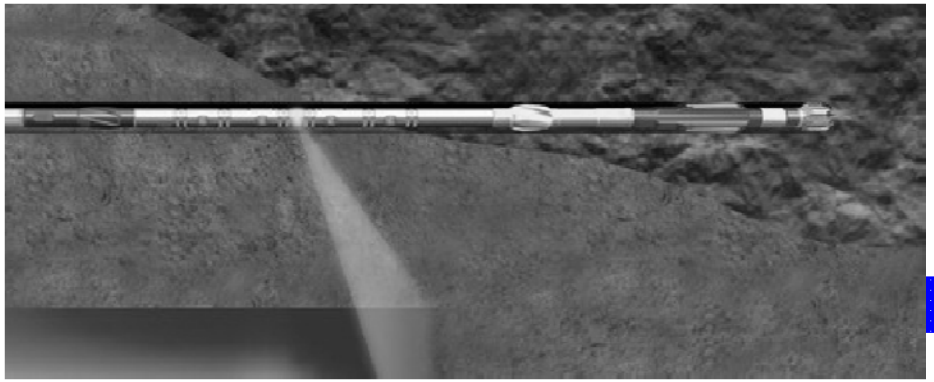


How Can We Quantify That MSA Validates a Tighter Error Model?

Weighting Function	Basic model		Prop. Mode
Sensors			
ABX	0.0004 g		
ABY	0.0004 g		S
ABZ	0.0004 g		S
ASX	0.0005		S
ASY	0.0005		S
ASZ	0.0005		S
MBX	~~70 nT~~	35 nT	S
MBY	~~70 nT~~	35 nT	S
MBZ	~~70 nT~~	35 nT	S
MSX	~~0.0016~~	0.0008	S
MSY	~~0.0016~~	0.0008	S
MSZ	~~0.0016~~	0.0008	S

Target Error Model





Multi-Station Analysis

Calculation of the Post-Fit Covariance Matrix

Survey Measurements

$$\boldsymbol{P} = E\left[\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^T \middle| \boldsymbol{y}\right]$$

Post-Fit State Error Covariance Matrix

Partitioned State Vector

$$\boldsymbol{x} = \begin{bmatrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \hline \text{MFI} \\ \text{MDI} \\ \text{MSZ} \end{bmatrix} \begin{matrix} \} \boldsymbol{x}_x \\ \\ \\ \\ \\ \\ \} \boldsymbol{x}_c \\ \\ \end{matrix}$$

Compact Notation

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_x \\ \boldsymbol{x}_c \end{bmatrix}$$

Optimal (Minimum Variance) Estimate

Appropriately-Weighted Least Squares Performance Index:

$$J = (\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{x}})^T \boldsymbol{R}^{-1} (\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{x}})$$

State Estimate

Random Meas. Noise:

$$E[\boldsymbol{\epsilon}] = \boldsymbol{0}$$
$$E[\boldsymbol{\epsilon}\boldsymbol{\epsilon}^T] = \boldsymbol{R}$$

Linearized survey measurement equation:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{\epsilon}$$

True State

Solution Via Orthogonal Transformations

$$J = (\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{x}})^T \left(\boldsymbol{R}^{-\frac{1}{2}}\right)^T \boldsymbol{Q}^T \boldsymbol{Q} \boldsymbol{R}^{-\frac{1}{2}} (\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{x}})$$

$$= \left\| \boldsymbol{Q}\boldsymbol{R}^{-\frac{1}{2}} (\boldsymbol{H}\boldsymbol{x} - \boldsymbol{y}) \right\|^2$$

$\boldsymbol{Q}^T\boldsymbol{Q} = \boldsymbol{I}$ (Orthogonal Matrix)

Choose Q So That:

$$\boldsymbol{Q}\boldsymbol{R}^{-\frac{1}{2}}\boldsymbol{H} = \begin{bmatrix} \boldsymbol{U}_{n\times n} \\ \boldsymbol{0}_{(m-n)\times n} \end{bmatrix}$$

$$\boldsymbol{Q}\boldsymbol{R}^{-\frac{1}{2}}\boldsymbol{y} = \begin{bmatrix} \boldsymbol{b}_{n\times 1} \\ \boldsymbol{e}_{(m-n)\times 1} \end{bmatrix}$$

Substitution yields

$$J = \left\| \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{0} \end{bmatrix} \hat{\boldsymbol{x}} - \begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{e} \end{bmatrix} \right\|^2$$

$$= \|\boldsymbol{U}\hat{\boldsymbol{x}} - \boldsymbol{b}\|^2 + \|\boldsymbol{e}\|^2$$

Solution & Post-Fit State Covariance Matrix:

$$\hat{\boldsymbol{x}} = \boldsymbol{U}^{-1}\boldsymbol{b}$$

$$\boldsymbol{P} = \boldsymbol{U}^{-1}\boldsymbol{U}^{-T}$$

Consider Covariance Analysis (Partition the Problem)

$$J = \left\| \begin{bmatrix} \boldsymbol{U}_{xx} & \boldsymbol{U}_{xc} \\ \boldsymbol{0} & \boldsymbol{U}_{cc} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{x}}_x \\ \bar{\boldsymbol{x}}_c \end{bmatrix} - \begin{bmatrix} \boldsymbol{b}_x \\ \boldsymbol{b}_c \end{bmatrix} \right\|^2 + \|\boldsymbol{e}\|^2$$

A Priori Consider State Estimate (Fixed)

To Minimize J, Solve

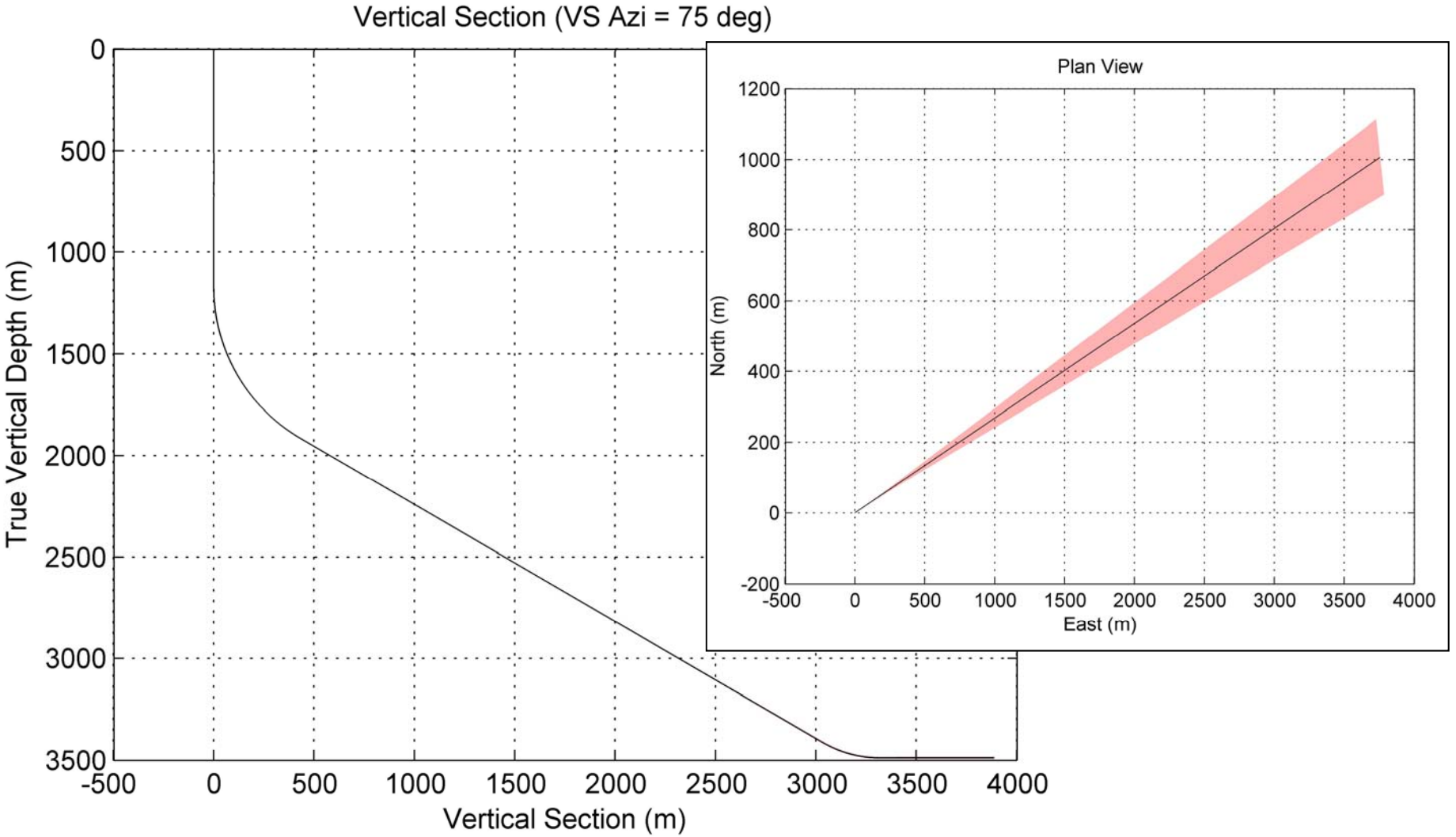
$$\boldsymbol{U}_{xx}\hat{\boldsymbol{x}}_x + \boldsymbol{U}_{xc}\bar{\boldsymbol{x}}_c = \boldsymbol{b}_x \quad \Longrightarrow \quad \begin{aligned} \hat{\boldsymbol{x}}_x &= \boldsymbol{U}_{xx}^{-1}\boldsymbol{b}_x - \boldsymbol{U}_{xx}^{-1}\boldsymbol{U}_{xc}\bar{\boldsymbol{x}}_c \\ &= \boldsymbol{U}_{xx}^{-1}\boldsymbol{b}_x + \boldsymbol{S}\bar{\boldsymbol{x}}_c \end{aligned}$$

Post-Fit State Covariance Matrix:

$$\boldsymbol{P} = E\begin{bmatrix} \left(\boldsymbol{U}_{xx}^{-1}\boldsymbol{U}_{xx}^{-T} + \boldsymbol{S}\bar{\boldsymbol{P}}_{cc}\boldsymbol{S}^T\right) & -\boldsymbol{S}\bar{\boldsymbol{P}}_{cc} \\ -\bar{\boldsymbol{P}}_{cc}\boldsymbol{S}^T & \bar{\boldsymbol{P}}_{cc} \end{bmatrix}$$

A Priori Consider State Covariance Matrix

Analysis of ISCWSA Test Well #1 (North Sea Extended Reach Well)



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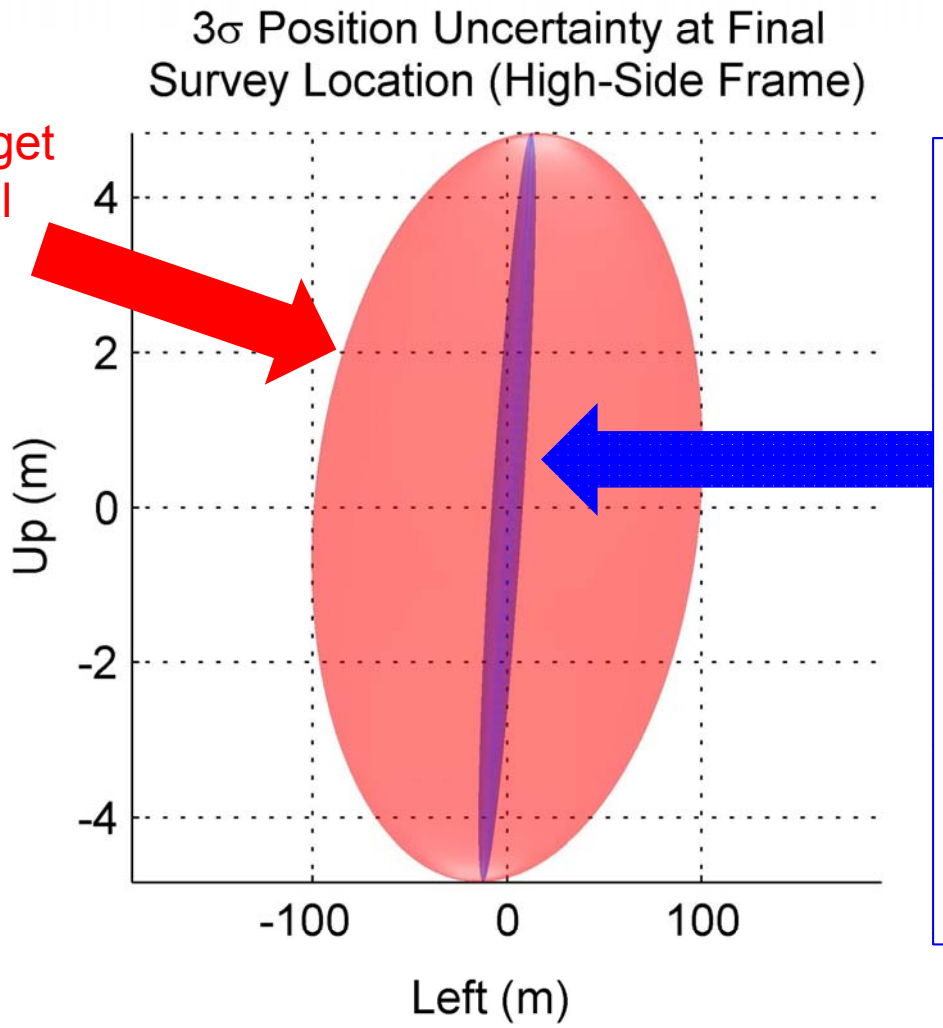
Analysis of ISCWSA Test Well #1 (North Sea Extended Reach Well)

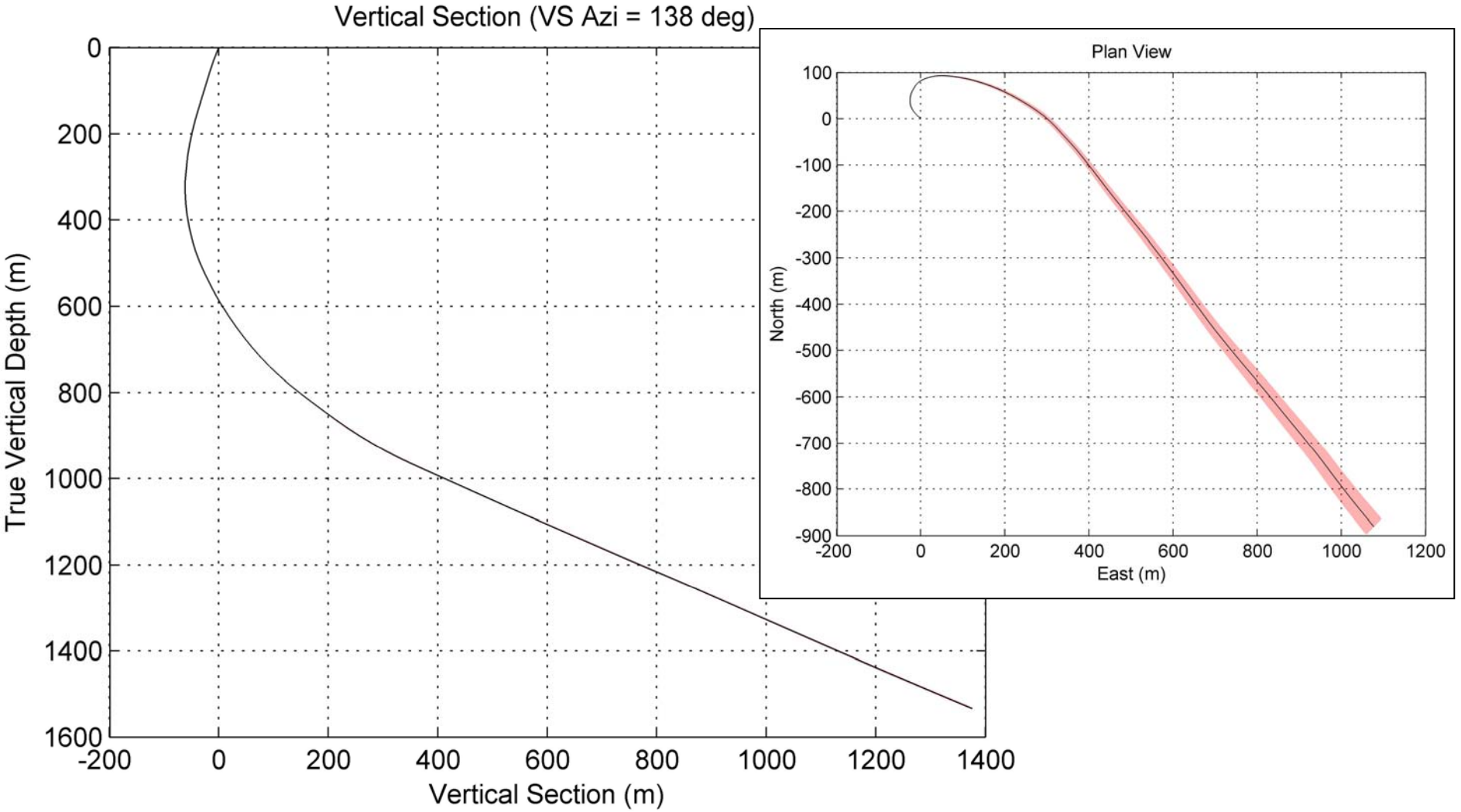
Tighter Target Error Model



$$\boldsymbol{P} = E\begin{bmatrix} (\boldsymbol{U}_{xx}^{-1}\boldsymbol{U}_{xx}^{-T} + \boldsymbol{S}\bar{\boldsymbol{P}}_{cc}\boldsymbol{S}^T) & -\boldsymbol{S}\bar{\boldsymbol{P}}_{cc} \\ -\bar{\boldsymbol{P}}_{cc}\boldsymbol{S}^T & \bar{\boldsymbol{P}}_{cc} \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \text{MSZ} \\ \hline \text{MFI} \\ \text{MDI} \end{bmatrix} \begin{matrix} \} \\ \\ \\ \\ \\ \\ \\ \} \end{matrix} \begin{matrix} \boldsymbol{x}_x \\ \\ \\ \\ \\ \\ \\ \boldsymbol{x}_c \end{matrix}$$

Analysis of an Actual Well



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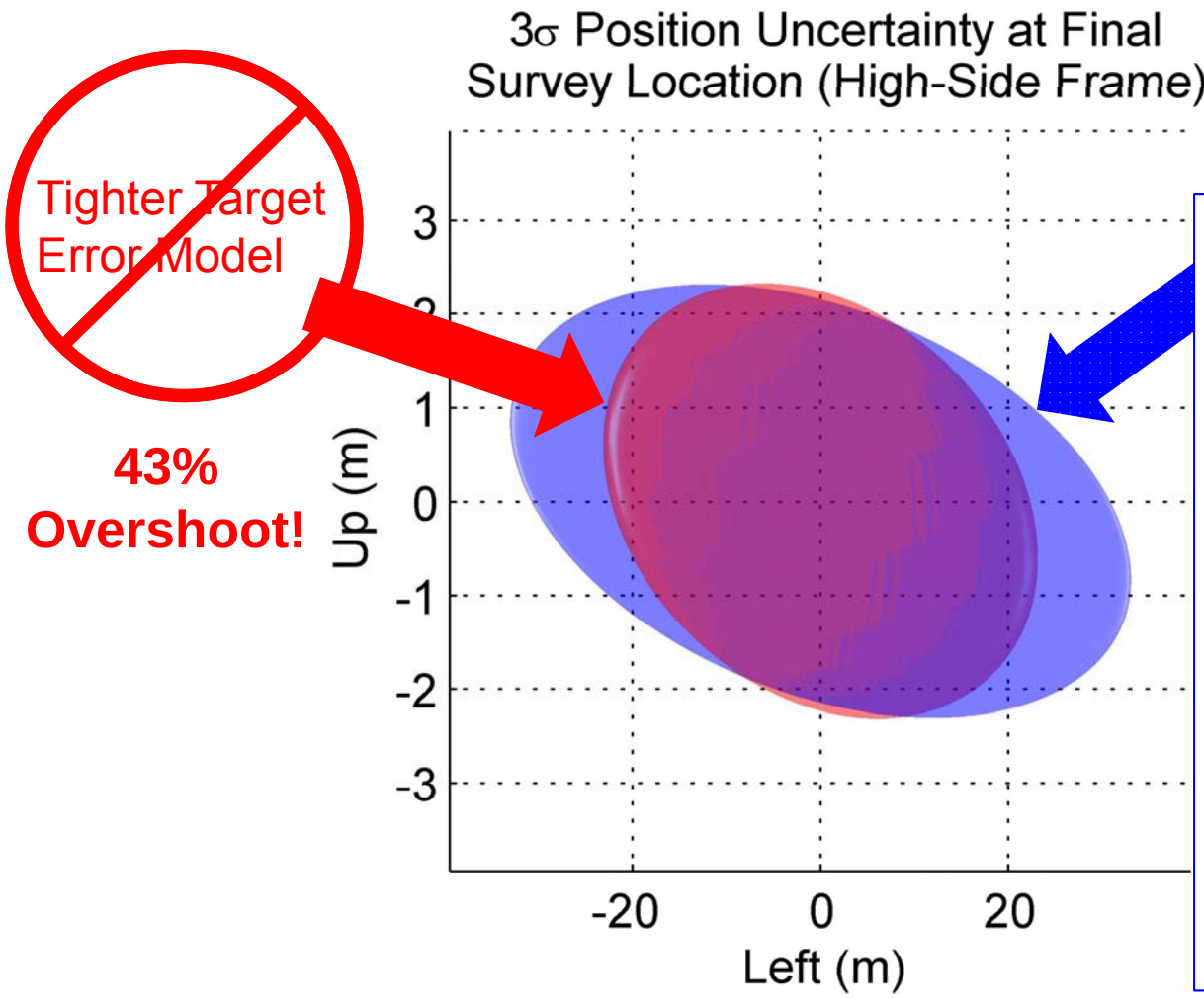
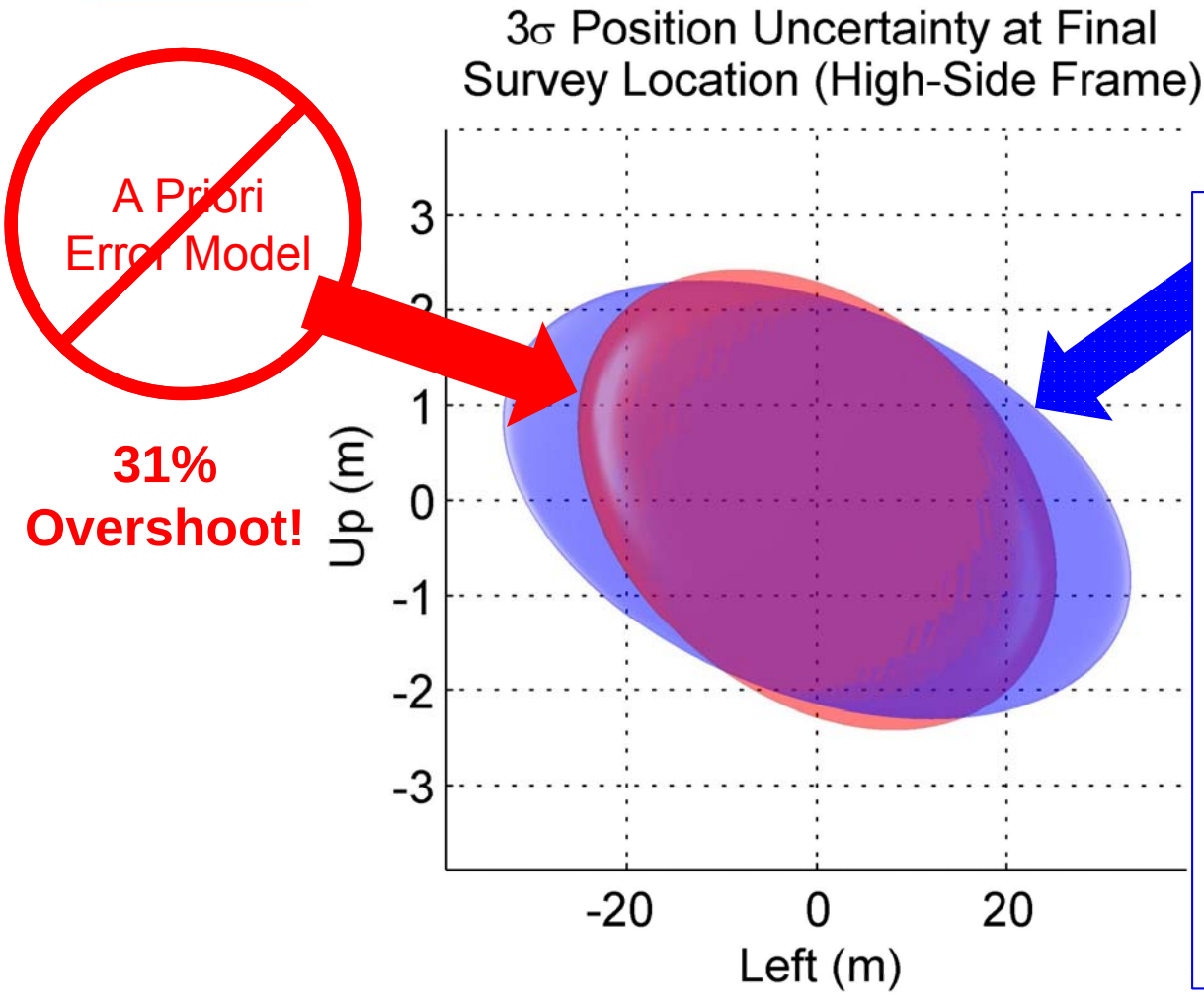
Analysis of an Actual Well: Tighter Target EM



$$\boldsymbol{P} = E\begin{bmatrix} (\boldsymbol{U}_{xx}^{-1}\boldsymbol{U}_{xx}^{-T} + \boldsymbol{S}\bar{\boldsymbol{P}}_{cc}\boldsymbol{S}^T) & -\boldsymbol{S}\bar{\boldsymbol{P}}_{cc} \\ -\bar{\boldsymbol{P}}_{cc}\boldsymbol{S}^T & \bar{\boldsymbol{P}}_{cc} \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \text{MSZ} \\ \hline \text{MFI} \\ \text{MDI} \end{bmatrix} \begin{matrix} \} \\ \\ \\ \\ \\ \\ \\ \} \end{matrix} \begin{matrix} \boldsymbol{x}_x \\ \\ \\ \\ \\ \\ \\ \boldsymbol{x}_c \end{matrix}$$

Analysis of an Actual Well: A Priori EM



$$\boldsymbol{P} = E\begin{bmatrix} (\boldsymbol{U}_{xx}^{-1}\boldsymbol{U}_{xx}^{-T} + \boldsymbol{S}\bar{\boldsymbol{P}}_{cc}\boldsymbol{S}^T) & -\boldsymbol{S}\bar{\boldsymbol{P}}_{cc} \\ -\bar{\boldsymbol{P}}_{cc}\boldsymbol{S}^T & \bar{\boldsymbol{P}}_{cc} \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \text{MSZ} \\ \hline \text{MFI} \\ \text{MDI} \end{bmatrix} \begin{matrix} \} \\ \\ \\ \\ \\ \\ \\ \} \end{matrix} \begin{matrix} \boldsymbol{x}_x \\ \\ \\ \\ \\ \\ \\ \boldsymbol{x}_c \end{matrix}$$

What about Axial Magnetic Interference?

Target Error Model

Weighting Function	Basic model		Prop. Mode
Sensors			
ABX	0.0004 g		S
ABY	0.0004 g		S
ABZ	0.0004 g		S
ASX	0.0005		S
ASY	0.0005		S
ASZ	0.0005		S
MBX	~~70 nT~~	35 nT	S
MBY	~~70 nT~~	35 nT	S
MBZ	~~70 nT~~	35 nT	S
MSX	~~0.0016~~	0.0008	S
MSY	~~0.0016~~	0.0008	S
MSZ	~~0.0016~~	0.0008	S
AMI	150 nT		S

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Target Error Model w/ AMI Reduction

Weighting Function	Basic model		Prop. Mode
Sensors			
ABX	0.0004 g		S
ABY	0.0004 g		S
ABZ	0.0004 g		S
ASX	0.0005		S
ASY	0.0005		S
ASZ	0.0005		S
MBX	~~70 nT~~	35 nT	S
MBY	~~70 nT~~	35 nT	S
MBZ	~~70 nT~~	35 nT	S
MSX	~~0.0016~~	0.0008	S
MSY	~~0.0016~~	0.0008	S
MSZ	~~0.0016~~	0.0008	S
AMI	~~150 nT~~	75 nT	S

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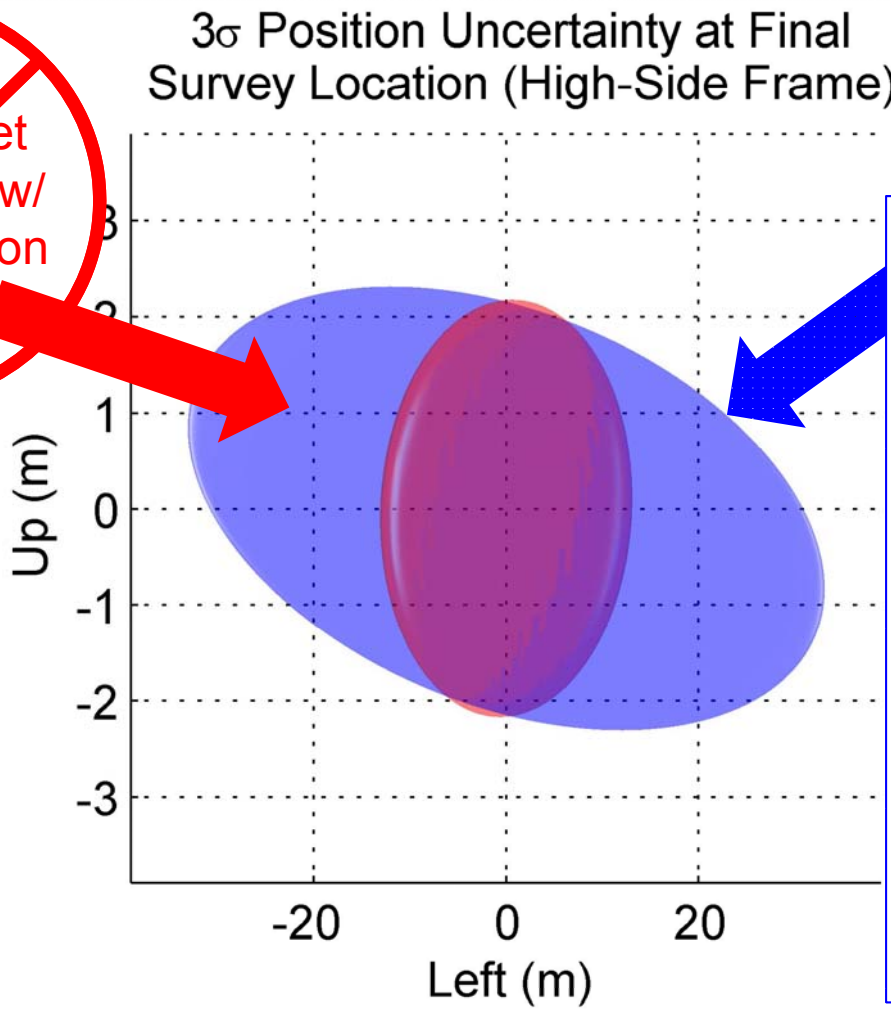
Analysis of an Actual Well: Tighter Target EM (Including Reduced Axial Magnetic Interference)

Tighter Target Error Model w/ AMI Reduction

150% Overshoot!



$$\boldsymbol{P} = E\begin{bmatrix} (\boldsymbol{U}_{xx}^{-1}\boldsymbol{U}_{xx}^{-T} + \boldsymbol{S}\bar{\boldsymbol{P}}_{cc}\boldsymbol{S}^T) & -\boldsymbol{S}\bar{\boldsymbol{P}}_{cc} \\ -\bar{\boldsymbol{P}}_{cc}\boldsymbol{S}^T & \bar{\boldsymbol{P}}_{cc} \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \text{MSZ} \\ \hline \text{MFI} \\ \text{MDI} \end{bmatrix} \begin{matrix} \} \\ \\ \\ \\ \\ \\ \\ \} \end{matrix} \begin{matrix} \boldsymbol{x}_x \\ \\ \\ \\ \\ \\ \\ \boldsymbol{x}_c \end{matrix}$$

Questions?

