

When is it Valid to Assign a Tighter Error Model to MSA-Processed Data?

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A Priori and Target Error Models

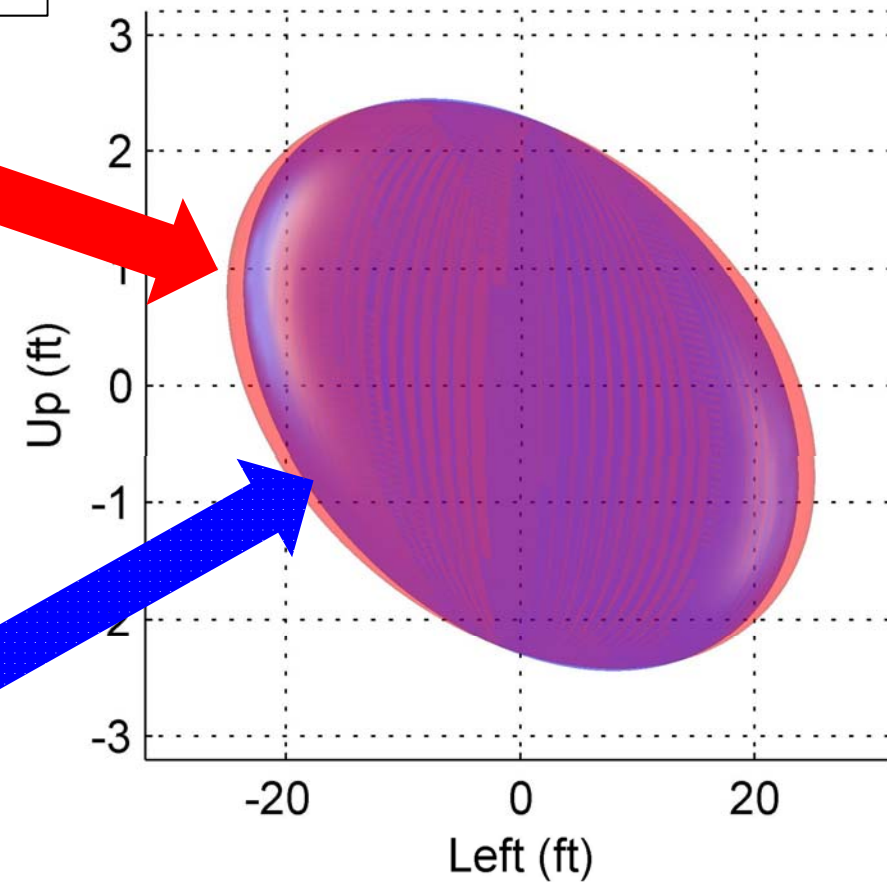
A Priori Error Model			Target Error Model		
Weighting Function	Basic model	Prop. Mode	Weighting Function	Basic model	Prop. Mode
Sensors			Sensors		
ABX	0.0004 g	S	ABX	0.0004 g	S
ABY	0.0004 g	S	ABY	0.0004 g	S
ABZ	0.0004 g	S	ABZ	0.0004 g	S
ASX	0.0005	S	ASX	0.0005	S
ASY	0.0005	S	ASY	0.0005	S
ASZ	0.0005	S	ASZ	0.0005	S
MBX	70 nT	S	MBX	70 nT 35 nT	S
MBY	70 nT	S	MBY	70 nT 35 nT	S
MBZ	70 nT	S	MBZ	70 nT 35 nT	S
MSX	0.0016	S	MSX	0.0016 0.0008	S
MSY	0.0016	S	MSY	0.0016 0.0008	S
MSZ	0.0016	S	MSZ	0.0016 0.0008	S
AMI	150 nT	S	AMI	150 nT	S
	⋮			⋮	

What Defines a Valid Error Model?

Weighting Function	Basic model	Prop. Mode
Sensors		
ABX	0.0004 g	
ABY	0.0004 g	S
ABZ	0.0004 g	S
ASX	0.0005	S
ASY	0.0005	S
ASZ	0.0005	S
MBX	70 nT	S
MBY	70 nT	S
MBZ	70 nT	S
MSX	0.0016	S
MSY	0.0016	S
MSZ	0.0016	S

A Priori Error Model

3 σ Position Uncertainty at Survey Location (High-Side Frame)

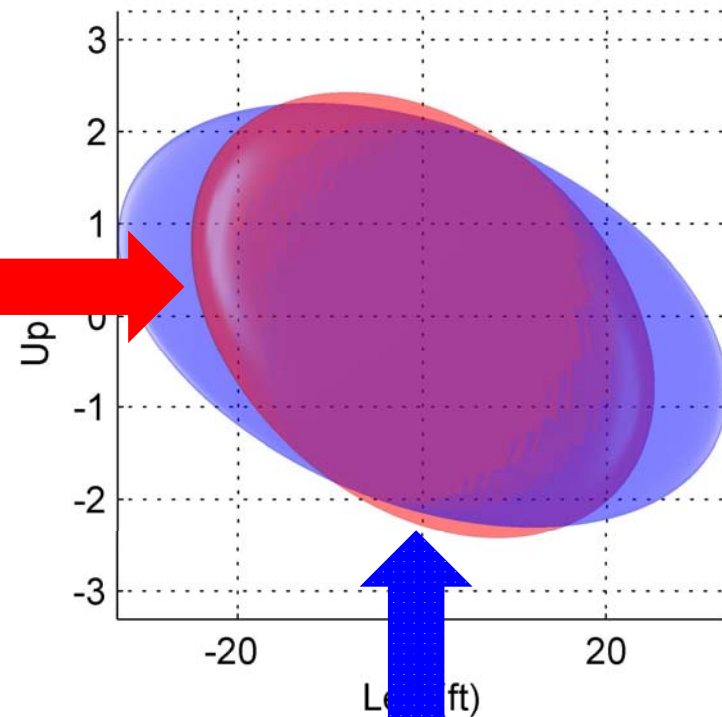


How Can We Quantify That MSA Validates a Tighter Error Model?

Weighting Function	Basic model	Prop. Mode
Sensors		
ABX	0.0004 g	
ABY	0.0004 g	S
ABZ	0.0004 g	S
ASX	0.0005	S
ASY	0.0005	S
ASZ	0.0005	S
MBX	70 nT 35 nT	S
MBY	70 nT 35 nT	S
MBZ	70 nT 35 nT	S
MSX	0.0016 0.0008	S
MSY	0.0016 0.0008	S
MSZ	0.0016 0.0008	S

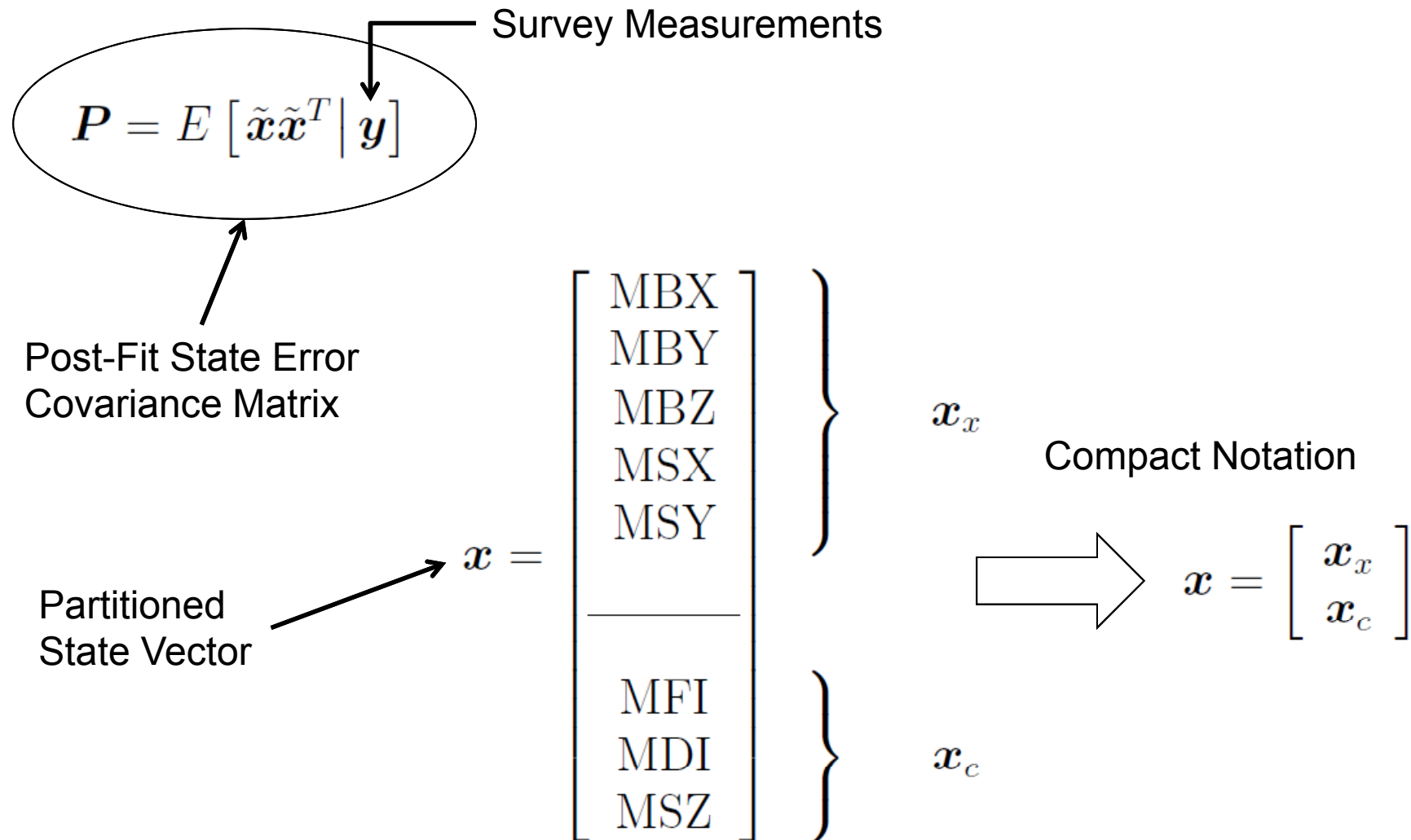
Target Error Model

3 σ Position Uncertainty at Survey Location (High-Side Frame)



Multi-Station Analysis

Calculation of the Post-Fit Covariance Matrix



Optimal (Minimum Variance) Estimate

Appropriately-Weighted Least Squares
Performance Index:

$$J = (\mathbf{y} - \mathbf{H}\hat{\mathbf{x}})^T \mathbf{R}^{-1} (\mathbf{y} - \mathbf{H}\hat{\mathbf{x}})$$

State Estimate

Linearized survey measurement
equation:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\epsilon}$$

True State

Random Meas.
Noise:

$$\begin{aligned} E[\boldsymbol{\epsilon}] &= \mathbf{0} \\ E[\boldsymbol{\epsilon}\boldsymbol{\epsilon}^T] &= \mathbf{R} \end{aligned}$$

Solution Via Orthogonal Transformations

$$J = (\mathbf{y} - \mathbf{H}\hat{\mathbf{x}})^T \left(\mathbf{R}^{-\frac{1}{2}} \right)^T \mathbf{Q}^T \mathbf{Q} \mathbf{R}^{-\frac{1}{2}} (\mathbf{y} - \mathbf{H}\hat{\mathbf{x}})$$

$$= \left\| \mathbf{Q} \mathbf{R}^{-\frac{1}{2}} (\mathbf{H}\hat{\mathbf{x}} - \mathbf{y}) \right\|^2$$

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \quad (\text{Orthogonal Matrix})$$

Choose Q So That:

$$\mathbf{Q} \mathbf{R}^{-\frac{1}{2}} \mathbf{H} = \begin{bmatrix} \mathbf{U}_{n \times n} \\ \mathbf{0}_{(m-n) \times n} \end{bmatrix}$$

$$\mathbf{Q} \mathbf{R}^{-\frac{1}{2}} \mathbf{y} = \begin{bmatrix} \mathbf{b}_{n \times 1} \\ \mathbf{e}_{(m-n) \times 1} \end{bmatrix}$$

Substitution yields

$$J = \left\| \begin{bmatrix} \mathbf{U} \\ \mathbf{0} \end{bmatrix} \hat{\mathbf{x}} - \begin{bmatrix} \mathbf{b} \\ \mathbf{e} \end{bmatrix} \right\|^2$$

$$= \|\mathbf{U}\hat{\mathbf{x}} - \mathbf{b}\|^2 + \|\mathbf{e}\|^2$$

Solution & Post-Fit State

Covariance Matrix:

$$\hat{\mathbf{x}} = \mathbf{U}^{-1} \mathbf{b}$$

$$\mathbf{P} = \mathbf{U}^{-1} \mathbf{U}^{-T}$$

Consider Covariance Analysis (Partition the Problem)

$$J = \left\| \begin{bmatrix} U_{xx} & U_{xc} \\ \mathbf{0} & U_{cc} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}}_x \\ \bar{\mathbf{x}}_c \end{bmatrix} - \begin{bmatrix} \mathbf{b}_x \\ \mathbf{b}_c \end{bmatrix} \right\|^2 + \|\mathbf{e}\|^2$$

↑ A Priori Consider State Estimate (Fixed)

To Minimize J, Solve

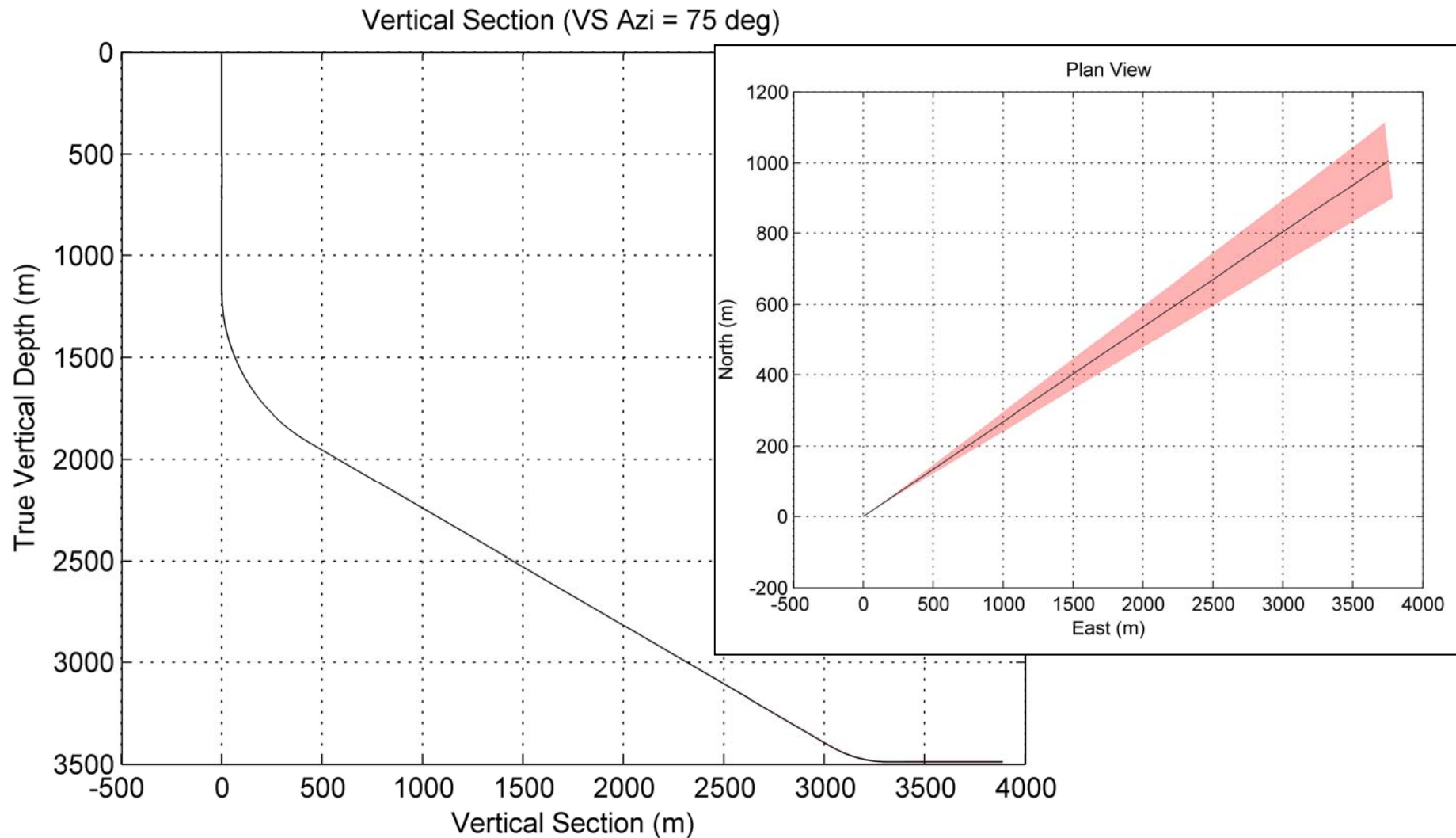
$$U_{xx}\hat{\mathbf{x}}_x + U_{xc}\bar{\mathbf{x}}_c = \mathbf{b}_x \quad \longrightarrow \quad \begin{aligned} \hat{\mathbf{x}}_x &= U_{xx}^{-1}\mathbf{b}_x - U_{xx}^{-1}U_{xc}\bar{\mathbf{x}}_c \\ &= U_{xx}^{-1}\mathbf{b}_x + \mathbf{S}\bar{\mathbf{x}}_c \end{aligned}$$

Post-Fit State Covariance Matrix:

$$\mathbf{P} = \mathbf{E} \begin{bmatrix} (U_{xx}^{-1}U_{xx}^{-T} + \mathbf{S}\bar{\mathbf{P}}_{cc}\mathbf{S}^T) & -\mathbf{S}\bar{\mathbf{P}}_{cc} \\ -\bar{\mathbf{P}}_{cc}\mathbf{S}^T & \bar{\mathbf{P}}_{cc} \end{bmatrix}$$

↑ A Priori Consider State Covariance Matrix

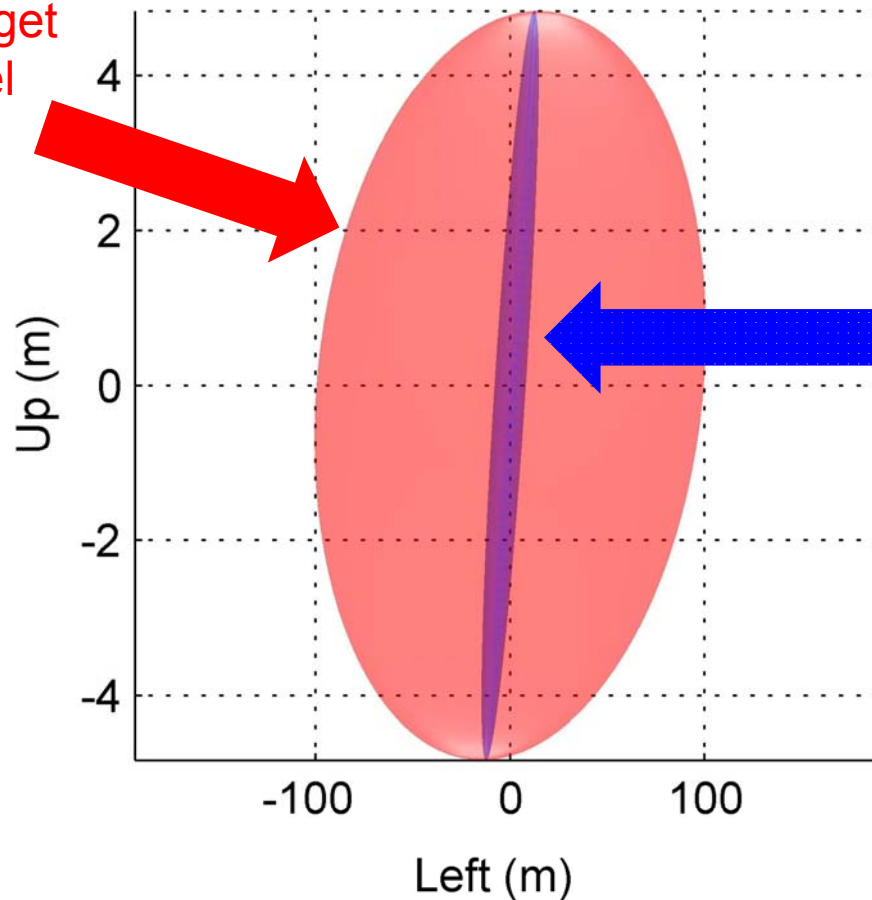
Analysis of ISCWSA Test Well #1 (North Sea Extended Reach Well)



Analysis of ISCWSA Test Well #1 (North Sea Extended Reach Well)

3 σ Position Uncertainty at Final Survey Location (High-Side Frame)

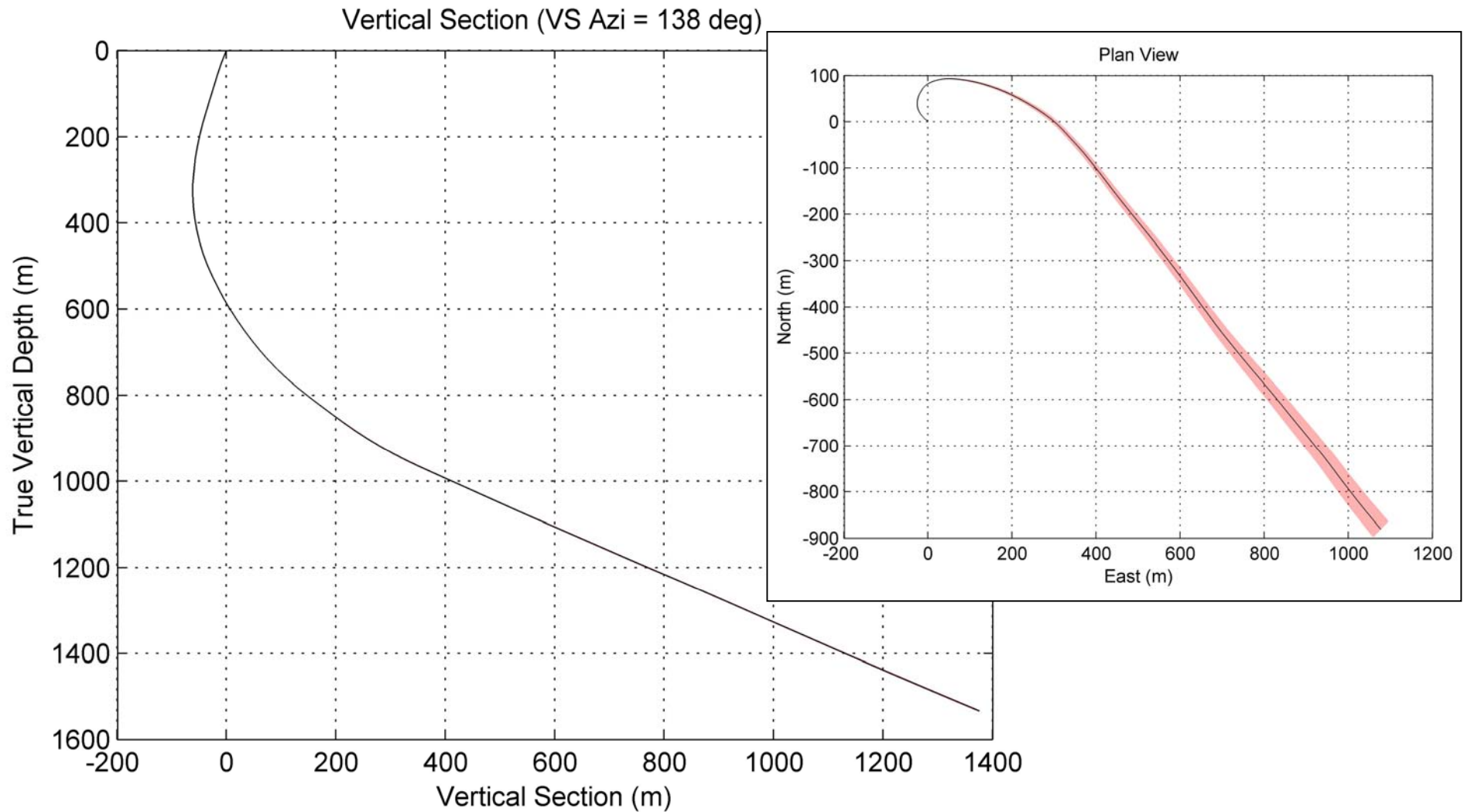
Tighter Target Error Model



$$P = E \begin{bmatrix} (U_{xx}^{-1}U_{xx}^{-T} + S\bar{P}_{cc}S^T) & -S\bar{P}_{cc} \\ -\bar{P}_{cc}S^T & \bar{P}_{cc} \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \text{MSZ} \\ \hline \text{MFI} \\ \text{MDI} \end{bmatrix} \left. \begin{array}{l} \vphantom{\begin{matrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \text{MSZ} \end{matrix}} \\ \vphantom{\begin{matrix} \text{MFI} \\ \text{MDI} \end{matrix}} \end{array} \right\} \begin{array}{l} \mathbf{x}_x \\ \\ \mathbf{x}_c \end{array}$$

Analysis of an Actual Well

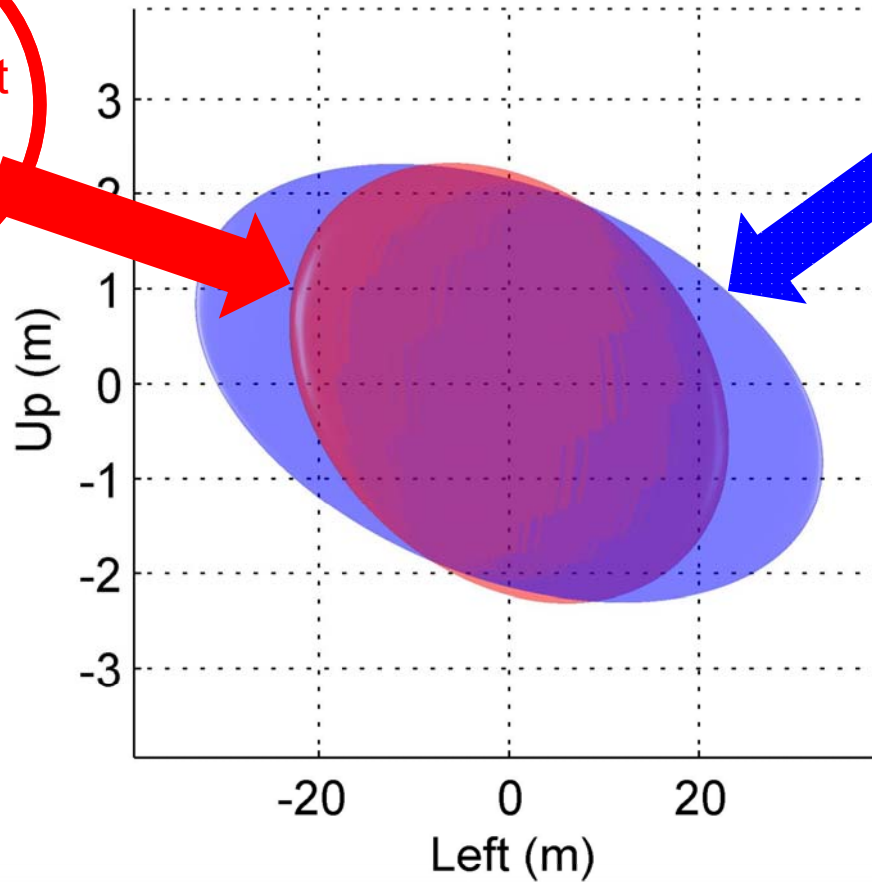


Analysis of an Actual Well: Tighter Target EM

3 σ Position Uncertainty at Final Survey Location (High-Side Frame)

~~Tighter Target Error Model~~

43% Overshoot!



$$P = E \begin{bmatrix} (U_{xx}^{-1}U_{xx}^{-T} + S\bar{P}_{cc}S^T) & -S\bar{P}_{cc} \\ -\bar{P}_{cc}S^T & \bar{P}_{cc} \end{bmatrix}$$

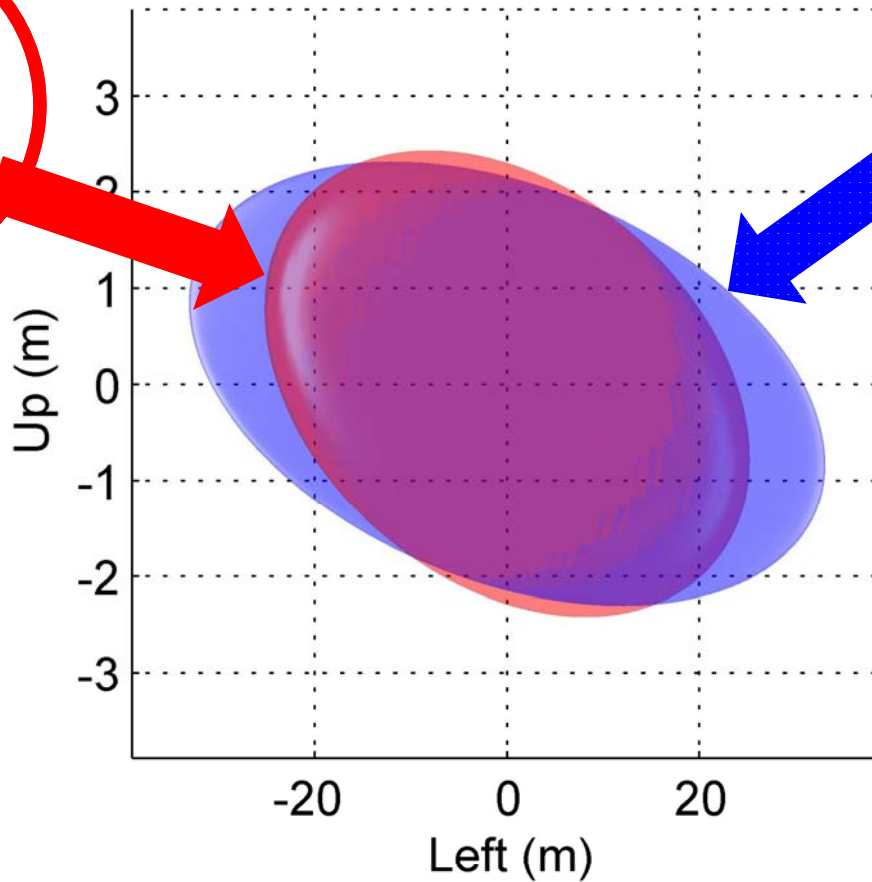
$$\mathbf{x} = \begin{bmatrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \text{MSZ} \\ \hline \text{MFI} \\ \text{MDI} \end{bmatrix} \begin{array}{l} \left. \vphantom{\begin{matrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \text{MSZ} \end{matrix}} \right\} \mathbf{x}_x \\ \left. \vphantom{\begin{matrix} \text{MFI} \\ \text{MDI} \end{matrix}} \right\} \mathbf{x}_c \end{array}$$

Analysis of an Actual Well: A Priori EM

3 σ Position Uncertainty at Final Survey Location (High-Side Frame)

~~A Priori Error Model~~

31% Overshoot!



$$P = E \begin{bmatrix} (U_{xx}^{-1}U_{xx}^{-T} + S\bar{P}_{cc}S^T) & -S\bar{P}_{cc} \\ -\bar{P}_{cc}S^T & \bar{P}_{cc} \end{bmatrix}$$

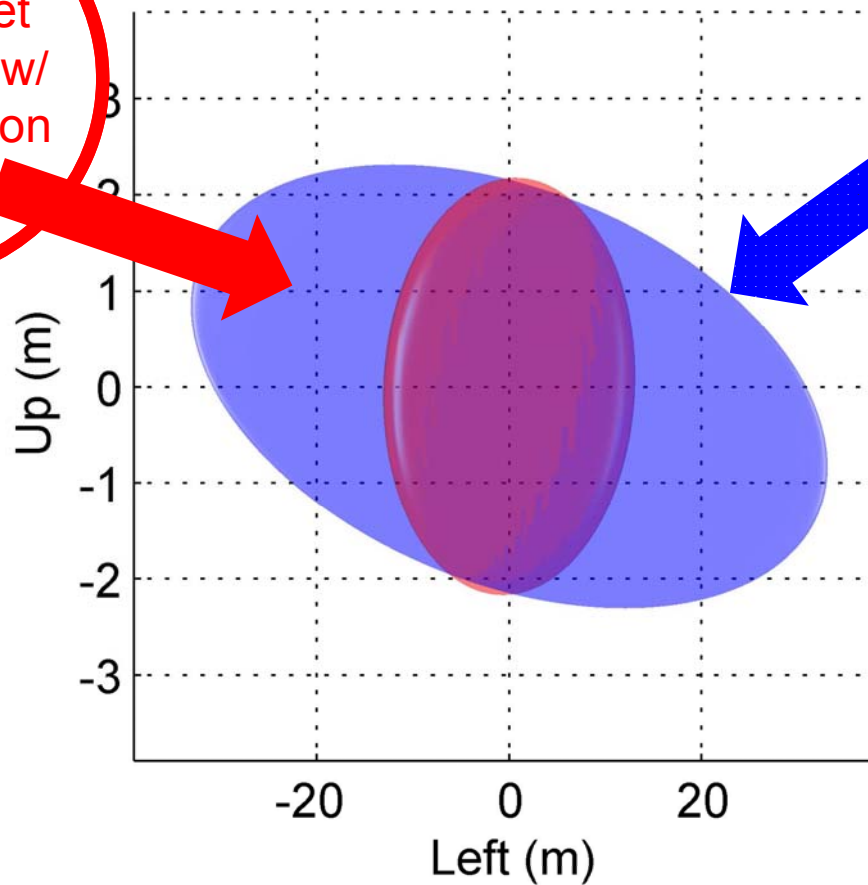
$$\mathbf{x} = \begin{bmatrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \text{MSZ} \\ \hline \text{MFI} \\ \text{MDI} \end{bmatrix} \begin{array}{l} \left. \vphantom{\begin{matrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \text{MSZ} \end{matrix}} \right\} \mathbf{x}_x \\ \left. \vphantom{\begin{matrix} \text{MFI} \\ \text{MDI} \end{matrix}} \right\} \mathbf{x}_c \end{array}$$

What about Axial Magnetic Interference?

Target Error Model			Target Error Model w/ AMI Reduction		
Weighting Function	Basic model	Prop. Mode	Weighting Function	Basic model	Prop. Mode
Sensors			Sensors		
ABX	0.0004 g	S	ABX	0.0004 g	S
ABY	0.0004 g	S	ABY	0.0004 g	S
ABZ	0.0004 g	S	ABZ	0.0004 g	S
ASX	0.0005	S	ASX	0.0005	S
ASY	0.0005	S	ASY	0.0005	S
ASZ	0.0005	S	ASZ	0.0005	S
MBX	70 nT 35 nT	S	MBX	70 nT 35 nT	S
MBY	70 nT 35 nT	S	MBY	70 nT 35 nT	S
MBZ	70 nT 35 nT	S	MBZ	70 nT 35 nT	S
MSX	0.0016 0.0008	S	MSX	0.0016 0.0008	S
MSY	0.0016 0.0008	S	MSY	0.0016 0.0008	S
MSZ	0.0016 0.0008	S	MSZ	0.0016 0.0008	S
AMI	150 nT	S	AMI	150 nT 75 nT	S
	⋮			⋮	

Analysis of an Actual Well: Tighter Target EM (Including Reduced Axial Magnetic Interference)

3 σ Position Uncertainty at Final Survey Location (High-Side Frame)



~~Tighter Target Error Model w/ AMI Reduction~~

150% Overshoot!

$$P = E \begin{bmatrix} (U_{xx}^{-1}U_{xx}^{-T} + S\bar{P}_{cc}S^T) & -S\bar{P}_{cc} \\ -\bar{P}_{cc}S^T & \bar{P}_{cc} \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \text{MSZ} \\ \hline \text{MFI} \\ \text{MDI} \end{bmatrix} \begin{array}{l} \left. \vphantom{\begin{matrix} \text{MBX} \\ \text{MBY} \\ \text{MBZ} \\ \text{MSX} \\ \text{MSY} \\ \text{MSZ} \end{matrix}} \right\} \mathbf{x}_x \\ \left. \vphantom{\begin{matrix} \text{MFI} \\ \text{MDI} \end{matrix}} \right\} \mathbf{x}_c \end{array}$$

Questions?