A New Look at Tool Misalignment

Jon Bang, Gyrodata Inc.

40th ISCWSA meeting, 30.10.2014, Amsterdam

Contents

» Background
» New misalignment model
» Comparison to existing models, and example calculations
» Conclusions

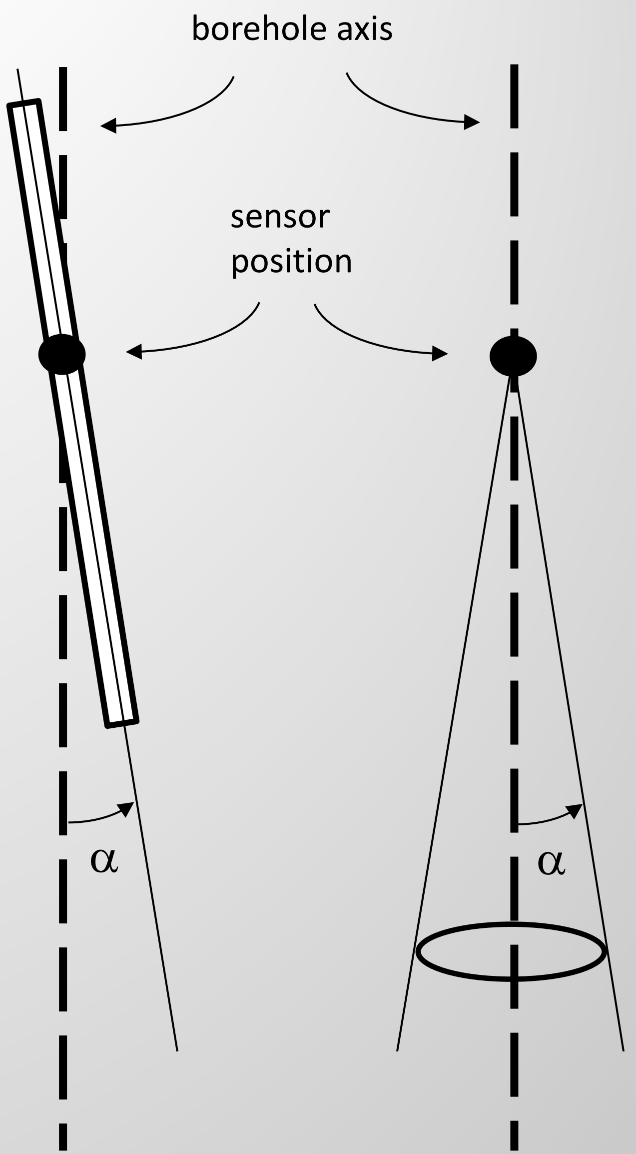
» Appendix: Mathematical details

Background

» Definition of tool misalignment α:
  – Angle between borehole axis and survey tool axis (local, at each survey station)

» Properties:
  – In general: unknown toolface
  – Error propagation: random or systematic between stations

» Analogous definition for sensor misalignment in tool, and misalignment between sensor axes



Background (cont.)

» The importance of tool misalignment:

- Affects all survey tools, and all survey operations
- High relative importance in top-hole sections, i.e., typically low-inclination wellbore sections
- Significant for long survey sections with fixed toolface (sliding tool)

Existing misalignment models

Origin	No. of inputs	Comments (+ / - indicate positive / negative properties)
Ekseth, PhD (1998)	2	Toolface dependency (-), weighting function singularity at vertical (-)
Brooks & Wilson, SPE 36863 (1996)	2	Toolface dependency (-), weighting function singularity at vertical (-)
Williamson, SPE 67616 (2000)		Adopted from Brooks & Wilson
Torkildsen et al., SPE 90408 (2008)	4	Toolface independent (+), multiple terms / alternatives (-), customised solution near vertical (singularity problem) (-)
New model	1	Direct physical foundation (+), toolface independent (+)*, valid for all directions including vertical and near vertical (+)

* Toolface used in derivation; final formulas are toolface-independent.

Introducing the new model

- Analysing misalignment in the D, I, A system (like all other error terms) is tempting, but leads to:
  - One physical error source modelled by several (2-4) «sources»
  - Customized or alternative solutions near vertical
  - The «vertical singularity» problem: $\delta A/\delta\alpha \sim 1/\sin(I)$
- However, the end results are variances and co-variances in the N, E, V system:
  - Can misalignment be analysed directly in N, E, V co-ordinates?
  - And would this solve any of the problems above?

Error propagation (matrix form)

Traditional model

mxy_1, mxy_2 ... (2-4 terms)

↓ (weighting functions)

dD, dI, dA vectors

↓ (co-ord. transf.)

dN, dE, dV vectors

↓

Var_N = cumulate $[dN*dN^T]$

Cov_{NE} = cumulate $[dN*dE^T]$

etc.

New model

misal (1 term)

↓

↓

↓

dN1, dN2, dE1, dE2, dV vectors

↓

Var_N = cumulate $[dN1*dN1^T + dN2*dN2^T]$

Cov_{NE} = cumulate $[dN1*dE1^T + dN2*dE2^T]$

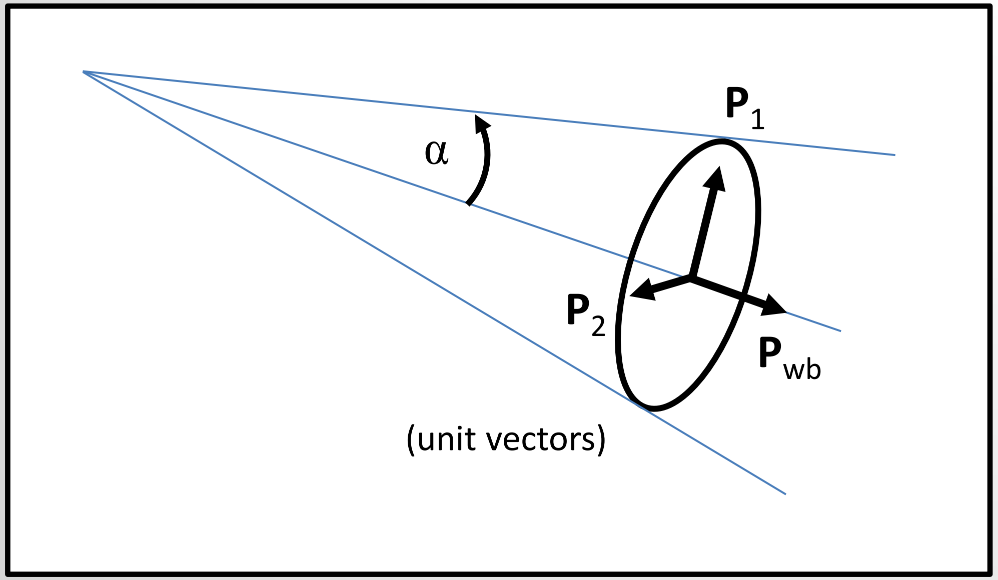
etc.

Single term
No weighting functions
No vertical singularity

Starting point for new model

- The position uncertainty due to misalignment α is always perpendicular to the (local) wellbore direction.
- At each measurement, the misalignment toolface angle τ is assumed uniform on [0° ... 360°] $\longrightarrow$ uncertainty «cone».
  - The toolface statistics is not related to the «random» or «systematic» nature of propagation between measurements.
- Consequently, the approach should be:
  1) Describe the uncertainty in the perpendicular plane (NEV system, and one τ).
  2) Average over τ.

A vector basis for the perpendicular plane



Choose $\mathbf{P}_1$ and $\mathbf{P}_2$ as an orthonormal basis for the wellbore's perp. plane.

<u>For example:</u>

$\mathbf{P}_1$ = high side = $\mathbf{P}_{wb}$ with $I \longrightarrow I+(\pi/2)$

$\mathbf{P}_2$ = lateral = $\mathbf{P}_{wb} \times \mathbf{P}_1$

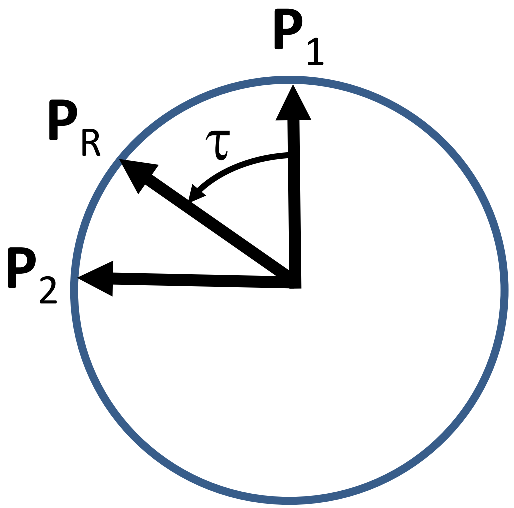
(results hold also for I = 0)

$$\mathbf{P}_{wb} = \begin{pmatrix} \sin(I)*\cos(A) \\ \sin(I)*\sin(A) \\ \cos(I) \end{pmatrix} \qquad \mathbf{P}_1 = \begin{pmatrix} \cos(I)*\cos(A) \\ \cos(I)*\sin(A) \\ -\sin(I) \end{pmatrix} \qquad \mathbf{P}_2 = \begin{pmatrix} -\sin(A) \\ \cos(A) \\ 0 \end{pmatrix}$$

Misalignment vector **R** in the perpendicular plane

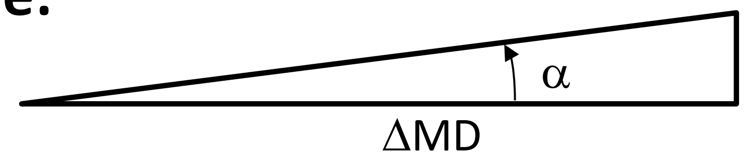
Direction:

(Looking towards the tool)



τ will be averaged over 360°, so reference direction (here: $\mathbf{P}_1$ = high side) is unimportant

Magnitude:



$m = |\mathbf{R}| = \Delta MD * \tan(\alpha)$

$$\mathbf{R} = m * \mathbf{P}_R = m * [\, \mathbf{P}_1 * \cos(\tau) + \mathbf{P}_2 * \sin(\tau)\,]$$

$$\mathbf{R} = \begin{pmatrix} dN_R \\ dE_R \\ dV_R \end{pmatrix} = m * \begin{pmatrix} \cos(I)*\cos(A)*\cos(\tau) & + & [-\sin(A)*\sin(\tau)] \\ \cos(I)*\sin(A)*\cos(\tau) & + & \cos(A)*\sin(\tau) \\ -\sin(I)*\cos(\tau) & + & 0 \end{pmatrix}$$

Averaging over toolface τ

(See Appendix for details)

Traditional model

mxy_1, mxy_2 ... (2-4 terms)

↓ (weighting functions)

dD, dI, dA vectors

↓ (co-ord. transf.)

dN, dE, dV vectors

↓

Var_N = cumulate [$dN*dN^T$]

Cov_{NE} = cumulate [$dN*dE^T$]

etc.

New model

misal (1 term)

↓

↓

↓

$dN(\tau)$, $dE(\tau)$, $dV(\tau)$ vectors

↓

Var_N = cumulate [$dN(\tau)*dN(\tau)^T$]

Cov_{NE} = cumulate [$dN(\tau)*dE(\tau)^T$]

etc.

(hand calc.)

Averages over toolface ...

... are incorporated here

Averaging over toolface τ

(See Appendix for details)

Traditional model

mxy_1, mxy_2 ... (2-4 terms)

↓ (weighting functions)

dD, dI, dA vectors

↓ (co-ord. transf.)

dN, dE, dV vectors

↓

Var_N = cumulate $[dN*dN^T]$

Cov_{NE} = cumulate $[dN*dE^T]$

etc.

New model

misal (1 term)

↓

↓

↓

dN1, dN2, dE1, dE2, dV vectors

↓

Var_N = cumulate $[dN1*dN1^T + dN2*dN2^T]$

Cov_{NE} = cumulate $[dN1*dE1^T + dN2*dE2^T]$

etc.

(hand calc.)

Averages over toolface ...

... are incorporated here

Variances and co-variances at station s (See Appendix for details)

$Var_N(s) = cumul_s [(\mathbf{dN1} * \mathbf{dN1}^T) + (\mathbf{dN2} * \mathbf{dN2}^T)]$

$Var_E(s) = cumul_s [(\mathbf{dE1} * \mathbf{dE1}^T) + (\mathbf{dE2} * \mathbf{dE2}^T)]$

$Var_V(s) = cumul_s [\mathbf{dV} * \mathbf{dV}^T]$

$Cov_{N,E}(s) = cumul_s [(\mathbf{dN1} * \mathbf{dE1}^T) + (\mathbf{dN2} * \mathbf{dE2}^T)]$

$Cov_{N,V}(s) = cumul_s [\mathbf{dN1} * \mathbf{dV}^T]$

$Cov_{E,V}(s) = cumul_s [\mathbf{dE1} * \mathbf{dV}^T]$

$dN1_j = (m_j/\sqrt{2}) * \cos(I_j) * \cos(A_j)$

$dN2_j = - (m_j/\sqrt{2}) * \sin(A_j)$

$dE1_j = (m_j/\sqrt{2}) * \cos(I_j) * \sin(A_j)$

$dE2_j = (m_j/\sqrt{2}) * \cos(A_j)$

$dV_j = - (m_j/\sqrt{2}) * \sin(I_j)$

$m_j = \Delta MD_j * \tan(\alpha_j)$

dN1 is the column vector $\begin{pmatrix} \vdots \\ dN1_j \\ \vdots \end{pmatrix}$

and $\mathbf{dN1}^T$ its transpose

«$cumul_s$» means cumulation of the matrix elements $\mathbf{dN1}*\mathbf{dN1}^T(j,k)$ etc. over the submatrix (1..s, 1..s):

Rotating tool (random misalignment): cumulate diagonal (j = k) only

Sliding tool (systematic misalignment): cumulate whole submatrix (all j, k)

How does this fit to existing methods?

» Resulting formulae are consistent with SPE 90408.

» Outputs are consistent with Compass.

» All necessary input is given in standard ipm files:

Present (MWD model)

Name	Vector	Tie-on	Unit	Magn.	Formula
w12	n	n	-	1.0	sin (I)
w34	n	n	-	1.0	cos(I)
mxy1	i	s	d	0.06	w12
mxy2	l	s	d	0.06	w12
mxy3	i	s	d	0.06	cos(A)*w34
mxy3	l	s	d	0.06	-sin(A)*w34
mxy4	i	s	d	0.06	sin(A)*w34
mxy4	l	s	d	0.06	cos(A)*w34

Future?

Name	Vector	Tie-on	Unit	Magn.(α)	Formula
misal	m	s	d	0.0849 (*)	1

(*) α = mxy value * $\sqrt{2}$

Any line contains
all information needed:
Tie-on, Unit, Magn. (*)

Example results

straight wellbore; $\alpha = 0.06\text{deg}*\sqrt{2}$; systematic

Wellbore	dMD	I (deg)	A (deg)	Var(N)	Var(E)	Var(V)	Cov(NE)	Cov(NV)	Cov(EV)
Vertical	3000	0	0	9.8696	9.8696	0	0	0	0
Vertical	3000	0	45	9.8696	9.8696	0	0	0	0
Vertical	3000	0	90	9.8696	9.8696	0	0	0	0
Vertical	3000	0	270	9.8696	9.8696	0	0	0	0
Slant	3000	30	0	7.4022	9.8696	2.4674	0	-4.2737	0
Slant	3000	45	0	4.9348	9.8696	4.9348	0	-4.9348	0
Slant	3000	60	0	2.4674	9.8696	7.4022	0	-4.2737	0
Horizontal	3000	90	0	0	9.8696	9.8696	0	0	0
Horizontal	3000	90	45	4.9348	4.9348	9.8696	-4.9348	0	0
Horizontal	3000	90	90	9.8696	0	9.8696	0	0	0
Horizontal	3000	90	270	9.8696	0	9.8696	0	0	0

Conclusions

- New representation of tool misalignment error α
  - Model based on physical origin
  - Described directly in NEV system
- Simple, and «universally» valid
  - Single term description, toolface independent results
  - No traditional weighting functions (by-passes DIA system)
  - Valid for any inclination and azimuth angles
  - In particular: no «vertical singularity»

Conclusions (cont.)

» Suited for error model implementation
  - Explicit equations for variances and co-variances are given
  - Uses only standard input, e.g. from ipm files (α = ipm value * $\sqrt{2}$)

» Helps to simplify error models
  - Easier understanding and communication of error models
  - Reduced risk for wrong application and results
  - Increased confidence in position uncertainty analysis

Thanks for helpful discussions:

Roger Ekseth, John Weston, Adrián Ledroz

Gyrodata Inc.

Thank you.

Appendix:
Calculation of variances and co-variances
(Mathematical details)

Summary

In the new model, NEV contributions are initially described as toolface-dependent (see expression for R, slide 10).

The toolface is eliminated from Var/Cov formulae by ensemble averaging (by hand). The results show how R can be modified.

=> Toolface-independent N1, N2, E1, E2, V + reformulation of Var and Cov.

The resulting formulae are implemented on computer.

=> Standard procedure for error propagation.

Error propagation (τ-dependent term)

Step 1: Calculate dN_R, dE_R, dV_R contributions along the wellbore

Step 2: Form variances/co-variances contributions at each station

Step 3: Average over (unknown) toolface τ

} by hand

Step 4: Cumulate per-station contributions, according to random or systematic nature of propagation of misalignment

Step 5: Sum variances/co-variances to contributions from other error terms

} computer

Step 1: dN_R, dE_R, dV_R contributions cumulated down to station s:

$\Sigma^s_{j=1} dN_R(\tau_j)$ etc. for E, V

Step 2: Variances/co-variances at station s:

$\Sigma^s_{j=1} dN_R(\tau_j) * \Sigma^s_{k=1} dN_R(\tau_k)$ etc. for N*E, ...

Step 3: Average over toolface:

- Cross-terms in dN*dN etc. form a matrix where each element (j,k) contains a product of $\cos(\tau_j)$ or $\sin(\tau_j)$ with $\cos(\tau_k)$ or $\sin(\tau_k)$.
- Since τ is unknown, the best estimate is the statistical mean, found by ensemble averaging over τ = 0°...360°.

Ensemble averages E{...} of cross product terms, over toolface τ:

Product terms	Random τ (= rotating tool)		Systematic τ (= sliding tool)	
	$j = k$	$j \neq k$	$j = k$	$j \neq k$
$E\{cos(\tau_j) * cos(\tau_k)\}$	**1/2**	**0**	**1/2**	**1/2**
$E\{cos(\tau_j) * sin(\tau_k)\}$	**0**	**0**	**0**	**0**
$E\{sin(\tau_j) * cos(\tau_k)\}$	**0**	**0**	**0**	**0**
$E\{sin(\tau_j) * sin(\tau_k)\}$	**1/2**	**0**	**1/2**	**1/2**

Observation 1:

Only [cos*cos] or [sin*sin] terms contribute, each by 1/2.

=> Discard product terms that contribute 0. For the remaining products: use original R vector terms with cos(τ) and sin(τ) replaced by $1/\sqrt{2}$.

Observation 2:

For random τ (rotating tool), only matrix diagonal elements (j=k) contribute.

For systematic τ (sliding tool), the whole matrix (all j, k) contribute.

=> Random or systematic propagation (step 4) is handled when summing matrix elements.

Resulting formulae on summation form

$$Var_N(s) = \Sigma^s_{k=1} \Sigma^s_{j=1} [(dN1_j * dN1_k) + (dN2_j * dN2_k)]$$

$$Var_E(s) = \Sigma^s_{k=1} \Sigma^s_{j=1} [(dE1_j * dE1_k) + (dE2_j * dE2_k)]$$

$$Var_V(s) = \Sigma^s_{k=1} \Sigma^s_{j=1} [dV_j * dV_k]$$

$$Cov_{N,E}(s) = \Sigma^s_{k=1} \Sigma^s_{j=1} [(dN1_j * dE1_k) + (dN2_j * dE2_k)]$$

$$Cov_{N,V}(s) = \Sigma^s_{k=1} \Sigma^s_{j=1} [dN1_j * dV_k]$$

$$Cov_{E,V}(s) = \Sigma^s_{k=1} \Sigma^s_{j=1} [dE1_j * dV_k]$$

s is the station number

$dN1_j = (m_j/\sqrt{2}) * \cos(I_j) * \cos(A_j)$

$dN2_j = - (m_j/\sqrt{2}) * \sin(A_j)$

$dE1_j = (m_j/\sqrt{2}) * \cos(I_j) * \sin(A_j)$

$dE2_j = (m_j/\sqrt{2}) * \cos(A_j)$

$dV_j = - (m_j/\sqrt{2}) * \sin(I_j)$

$m_j = \Delta MD_j * \tan(\alpha_j)$

Rotating tool (random misalignment): include j = k terms only
Sliding tool (systematic misalignment): include all j, k terms

Resulting formulae on matrix form

$Var_N(s) = cumul_s$ [(**dN1** * **dN1**^{T}) + (**dN2** * **dN2**^{T})]

$Var_E(s) = cumul_s$ [(**dE1** * **dE1**^{T}) + (**dE2** * **dE2**^{T})]

$Var_V(s) = cumul_s$ [**dV** * **dV**^{T}]

$Cov_{N,E}(s) = cumul_s$ [(**dN1** * **dE1**^{T}) + (**dN2** * **dE2**^{T})]

$Cov_{N,V}(s) = cumul_s$ [**dN1** * **dV**^{T}]

$Cov_{E,V}(s) = cumul_s$ [**dE1** * **dV**^{T}]

«$cumul_s$» means: cumulation of the matrix elements **dN1*dN1**^{T}(j,k) etc.
over the submatrix (1..s, 1..s):

Rotating tool (random misalignment): cumulate diagonal (j = k) only

Sliding tool (systematic misalignment): cumulate whole submatrix (all j, k)

s is the station number

dN1 is the column vector $\begin{pmatrix} \vdots \\ dN1_j \\ \vdots \end{pmatrix}$

and **dN1**^{T} its transpose

$dN1_j = (m_j/\sqrt{2}) * \cos(I_j) * \cos(A_j)$

$dN2_j = -(m_j/\sqrt{2}) * \sin(A_j)$

$dE1_j = (m_j/\sqrt{2}) * \cos(I_j) * \sin(A_j)$

$dE2_j = (m_j/\sqrt{2}) * \cos(A_j)$

$dV_j = -(m_j/\sqrt{2}) * \sin(I_j)$

$m_j = \Delta MD_j * \tan(\alpha_j)$